Discrete modeling of ultra-high-performance concrete with application to projectile penetration

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A B S T R A C T

In this paper, the Lattice Discrete Particle Model for fiber reinforced concrete (LDPM-F) is calibrated and validated with reference to a new high-strength, ultra-high-performance concrete (UHPC) named CORTUF and applied to the simulation of projectile penetration. LDPM-F is a three-dimensional model that simulates concrete at the length scale of coarse aggregate pieces (meso-scale) through the adoption of a discrete modeling framework for both fiber reinforcement and embedding matrix heterogeneity. In this study, CORTUF parameter identification is performed using basic laboratory fiber pull-out experiments and experiments relevant to a CORTUF mix without fiber reinforcement. Extensive comparisons of the numerical predictions against experimental data that were not used during the calibration phase (relevant to both plain CORTUF and CORTUF with fiber reinforcement) are used to validate the calibrated model and to provide solid evidence of its predictive capabilities. Simulations are then carried out to investigate the behavior of protective CORTUF panels subjected to projectile penetration, and the numerical results are discussed with reference to available experimental data obtained at the Engineering Research and Development Center (ERDC).

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1. Introduction

The design flexibility and natural durability of concrete help to make it the most used man-made material in the world. As opposed to other building materials, such as brick and stone, concrete has revolutionized previous construction methods with its intrinsic versatility and high compressive strength. Within the last century, a tremendous effort has been made to increase concrete strength. Over time, each new generation of the material became stronger and more durable, birthing the title we know today as ultra-high-performance concrete (UHPC). The material is sought for many uses including its adoption in high-rise structures, long-span bridges, offshore structures, and structural rehabilitation [1,2].

There are various methods used today to develop higher compressive strength in cementitious composites. One method is strengthening the interfacial transition zone (ITZ), the weak area surrounding the aggregate particle, which is often accomplished through a reduction of the aggregate size; this enhances homogeneity and reduces stress concentrations between the aggregate and mortar under loading. Another method to enhance the ITZ mechanical properties is reducing the water-to-binder ratio [3]. This process can be counterproductive if not balanced, since the high rates of hydration often result in autogeneous shrinkage and can lead to premature cracking during the setting phase [4]. Many high-performance concretes use silica fume, which acts like a microfiller and can also react with calcium hydroxide to increase the final strength of the composite [5]. Other methods reported in the literature that enhance the compressive strength of concrete are particle-size gradation for optimum packing of aggregates [6] and heat treatment [7–10].
Although there are many methods focused on increasing the compressive strength of concrete, techniques to increase tensile resistance and toughness are still in their infancy. One method typically used for this purpose is the addition of fibers in the matrix [5,11–14]. Multi-scale fiber reinforcing, in which fibers of various lengths and cross sectional sizes are used in the same mix, has also been explored in recent times [15]. When fiber reinforcing is used, the type and amount of fibers need to be optimized in order to maximize tensile strength and toughness while preserving workability and efficiency in terms of cost-to-performance ratio.

Enhancement of material toughness is particularly important in the case of the UHPC materials that are being tested to determine their efficiency against blast impact and penetration and for their use in particular areas susceptible to man-made and natural hazards [16,17]. As it pertains to projectile impact testing, many researchers found the penetration depth to increase with increasing impact velocities, and to decrease with increasing compressive strength. The perforation ballistic limit (the minimum velocity at which the bullet completely perforates the specimen) is typically reported to increase with higher compressive strength [18]. Aggregate size also plays an important role in material impact response. Stronger and larger sized aggregate particles improve impact resistance as long as enough workability is maintained [3,12,19]. For UHPC without fiber reinforcement, penetration events cause dynamic propagation of many micro-cracks and often lead to shattering of the target. On the contrary, inclusion of fibers confines cracks in the crater region in both front and rear faces and creates a more localized damage around the crater. However, for increasing fiber volume fraction, a saturation limit seems to exist after which no significant change is observed in both crater area and penetration depth [20–22].

When dealing with the effect of dynamic events on concrete structures it is also important to consider the strain-rate dependence of concrete response. Typical experimental observations on regular strength concrete report an increase of macroscopic mechanical properties such as, Young’s modulus, compressive strength, tensile strength, and fracture energy, for increasing strain rate (see, among many others, [23–25, 27–32]).

Evidences of strain-rate dependence of UHPC behavior are more scarce and contradictory compared to regular strength concrete. A few studies have been performed on the effect of strain rate on UHPCs that investigated the effect of fiber reinforcement and increasing strain-rate under tensile and compressive loadings. Some researchers found UHPCs with fibers to be less sensitive to strain-rate effects than similar strength unreinforced material [33,34]. Some other researchers [35] found no rate sensitivity for concrete reinforced with hooked steel fibers. Caverzan et al. [36] investigated the effect of temperature on strain rates of high-performance fiber-reinforced concrete (HPFRC) and found an increase in strength for increased strain rate up to a rate of 150 s$^{-1}$ for temperatures up to 200 °C, while material strength decreased for strain-rates above 150 s$^{-1}$ for higher temperatures due to fiber failure.

In most cases, rate effect is quantified through the Dynamic Increase Factor (DIF) defined as the ratio between the dynamic property of interest and the associated quasi-static value. The evolution of DIF with strain-rate is often approximated through a bilinear curve in which the second linear branch, starting for strain-rates around 1–10 s$^{-1}$, has a much steeper slope than the first branch. In many cases, the DIF curves are obtained by interpolating between experimental data showing huge scatter, especially for high strain-rates. Furthermore, it is common modeling procedure to equip concrete constitutive equations with rate dependence by using the DIF curve as a multiplicative factor for various model parameters. As some authors have shown recently [37–39], this approach is not correct in general because it does not distinguish between “intrinsic” phenomena, which should be included in the constitutive equations, and “apparent” phenomena, which are structural features of the response and should or should not be included in the constitutive equations depending on the spatial and temporal resolution of the adopted numerical model.

At least three intrinsic rate-dependent phenomena can be identified to affect concrete mechanical behavior: 1) creep, 2) Arrhenius-type behavior of fracture processes, and 3) contribution to the load carrying capacity of capillary and adsorbed water.

Concrete creep, the increase in time of deformation under constant applied load, is a complex phenomenon whose fundamental mechanism resides in the cement nano-structure, and it is hypothesized to be the result of shear slips among Calcium Silicate Hydrate (CШ) platelets [40,41]. Since the nano-structure of cement evolves as result of cement hydration and other chemical reactions, creep behavior changes over time: in general, young concrete features a much more pronounced viscous behavior compared to old concrete. In addition, concrete creep is affected by moisture content and temperature and their time rates [42]. In particular, the lower the relative humidity the less concrete creep deformation is typically observed. Wittmann [43] obtained an 87% decrease in creep deformation reducing the relative humidity from 100% to 10%. From the mathematical point of view, concrete creep is typically simulated by aging, temperature, and relative humidity dependent visco-elastic constitutive equations; this naturally leads to a dependence of the stress state on strain-rates.

However, creep cannot fully explain the fracturing behavior of concrete under dynamic loading. For example, creep cannot simulate experimental data on three point bending specimens showing almost instantaneous rehardening when subjected to a sudden increase of loading rate in the softening regime [44]. To model this phenomenon, Bàzent analyzed the fracture process within the classical theory of thermally activated processes, and by representing the frequency of bond rupture at the nano-scale with an Arrhenius equation, he derived a dependence of the cohesive stress (stress across a propagating cohesive crack) upon crack opening rate. This formulation was successfully used in Refs. [37,45,46] to simulate the dynamic behavior of concrete under low to moderate strain rates.

Furthermore, concrete is a porous material featuring a complex internal structure characterized by pore sizes spanning various order of magnitudes—from few nanometers to several millimeters. Depending upon the level of relative humidity, water is present in concrete pores in various forms (capillary, absorbed, or hindered) and, according to classical poro-mechanics concepts, it can certainly contribute to the overall carrying capacity. However, under quasi-static loading conditions the amount of load that pore water can carry is negligible because water is much more compressible than the solid skeleton. More complicated scenario arises under high confinement quasi-static triaxial loading because pore collapse occurs and water is squeezed out, even leading to a certain decrease in carrying capacity due to lubrication effects [47]. On the contrary, at high strain rates, water can contribute significantly to concrete strength. As far as tensile loading is concerned, some studies [48] have tried to explain the effect of water at high strain rates through the so-called Steffan effect [49] according to which the force needed to pull apart two parallel rigid plates containing a Newtonian fluid in between is proportional to the relative velocity of the two plates. This approach, although championed by some authors [50,51], has been criticized because its application to absorbed and hindered water (forming the majority of water in real concrete at relative humidity lower than 60–70%) is questionable. For triaxial compressive loading at high strain rates, water has a significant effect because, as pores collapse, water does not have enough time to be released leading to a
significant build up of pore pressure. Forquin and coworkers [47] report an increase of 55% of the hydrostatic stress measured in saturated concrete samples at a strain of 0.08 and a strain-rate of 120 s⁻¹ compared to dry samples.

While the intrinsic phenomena discussed above should (if relevant) be included in the formulation of appropriate constitutive equations for concrete, apparent phenomena should be filtered out from the experimental evidences and not considered in the constitutive equations unless they occur at a length scale smaller than the model resolution. The most important, and often misinterpreted, phenomenon leading to an apparent strain-rate dependence of concrete behavior is due to inertia effects. During dynamic events, stress waves travel through concrete and their path as well as their magnitude are affected by the presence of external boundaries as well as by concrete internal heterogeneity—this leads to “overt stresses” and fictitious confinement effects resulting in higher peak stresses erroneously interpreted as higher material strength. This aspect was analyzed in detail by the second author in an earlier publication [37]. As long as inertia is correctly accounted for in the interpretation of experimental results and in the numerical simulations, no inertia effects should be included in concrete constitutive equations, especially when they are formulated at fine length scales and capture material heterogeneity.

Another phenomenon associated with dynamic loading is the change in crack patterns. On one extreme, under quasi-static loading, a concrete bar in direct tension always fails with the propagation of one single macro-crack; on the other extreme, under ultra-high strain rates in which inertia effects are dominant, the same bar features a much more distributed crack pattern and comminution/pulverization might occur [52]. Also the microscopic characteristics [50,53] of crack patterns change with increasing strain-rate. For example, it has been observed that at high strain-rates, cracks initiate at the same time at multiple locations and in their subsequent propagation follow shorter paths of higher resistance (for example fracturing aggregate pieces) as opposed to more tortuous paths of least resistance (for example through the ITZ on the aggregate boundaries). Also, it is well reported for concrete [54] and other materials [55] the phenomenon of crack branching under dynamic crack propagation. Overall, the change in crack patterns leads to higher energy dissipation resulting in apparent strength and toughness increase for increasing strain rates.

2. Numerical modeling for UHPC materials and the LDPM-F formulation

Optimization of cementitious composites, with material properties tailored to a specific application, has been traditionally pursued through experimental investigations that are quite expensive and are not sustainable for investigation of a large number of different variables. A less expensive alternative is to optimize the material through numerical simulations.

Typical constitutive modeling for mode I fracture in concrete includes: (1) the cohesive crack model (CCM) [56,57], which simulates damage processes through the occurrence of displacement discontinuities, across which the ability to transfer stresses decreases as function of the discontinuity jump (crack opening); and (2) the crack band model [58,59], where cracks are assumed to be distributed over a certain band whose width is deemed a material property.

For more general triaxial stress states, strain-softening tensorial constitutive equations have been proposed in the literature [60–63], the most notable of which are the ones based on the microplane theory [64–68]. However, these type of models, suffer from pathological mesh sensitivity and spurious energy dissipation upon mesh refinement [69–71]. This problem has been somewhat addressed through the formulation of non-local models with the strain-based integral formulations, which allow simulation of micro-crack interaction and size effect [72–76].

A different class of models providing a better representation of the actual cracking phenomena occurring in the internal structure of the material is the one consisting of discrete approaches (lattice and particle models) in which concrete is discretized “a priori” according to its meso-structure. Earlier attempts in this direction are reported in Refs. [77–82], while more recent developments are discussed in Refs. [83,84].

Cusatis and coworkers formulated, calibrated, and validated a comprehensive discrete model for concrete, entitled Lattice Discrete Particle Model (LDPM) [85–87] and extended it to include the effect of fiber reinforcing [88,89]. The resulting model, labeled LDPM-F, demonstrated an unprecedented ability to model fiber-reinforced concrete and to correctly predict the toughening mechanisms due to the effect of fibers. For this reason, LDPM-F was selected in this study as the best candidate for the simulation of UHPC concrete materials in general, and CORTUF in particular.

LDPM represents concrete as a two-phase material composed of aggregate pieces held together by an embedding cementitious matrix. In the numerical representation, polyhedral shaped particles (Fig. 1a) simulate individual aggregate pieces (approximated as spheres) and their surrounding matrix. Geometry and topology of the model are obtained through: (1) random generation and placement of aggregate pieces; (2) delaunay tetrahedralization of aggregate centers; and (3) dual tessellation defining particle interfaces (triangular facets in Fig. 1a) assumed to be the location of potential cracks in the meso-structure. Fiber reinforcing is included in such a way to retain the discrete character of LDPM. All the fibers are randomly distributed in the material volume, and each fiber is associated with all intersecting meso-scale facets (Fig. 1b). Further details of the adopted fiber generation algorithm are discussed later in this paper.

LDPM describes meso-structure deformation through the adoption of rigid-body kinematics [85] for each individual particle. Based on this assumption and for given displacements and rotations of the particles associated with a given facet, the relative displacement \[\mathbf{u}_{\text{fi}}\] at the centroid of the facet can be used to define the following measures of strain.

\[
e_{N} = \frac{n^T\mathbf{u}_{\text{fi}}}{\mathbf{g}}; \quad e_{M} = \frac{m^T\mathbf{u}_{\text{fi}}}{\mathbf{g}}; \quad e_{L} = \frac{l^T\mathbf{u}_{\text{fi}}}{\mathbf{g}} \tag{1}
\]

where \(\mathbf{g}\) is the length of the straight line connecting two adjacent particle centers, and \(\mathbf{n}, \mathbf{m}, \mathbf{l}\) are unit vectors defining a local system of reference such that \(\mathbf{n}\) is orthogonal to the facet, and \(\mathbf{m}, \mathbf{l}\) are mutually orthogonal as well as tangential to the facet. Cusatis and Shauffert [90] showed that the strain definitions in Eq. (1) are consistent with the definition of strain in classical continuum mechanics. These strains can then be used to compute the LDPM facet stress tractions, \(\mathbf{t} = t_N\mathbf{n} + t_M\mathbf{m} + t_L\mathbf{l}\), in accordance with the LDPM constitutive law (Appendix A1). In addition, when a tensile facet deforms beyond the tensile elastic limit, the meso-scale crack opening can be calculated as \(\mathbf{w} = w_N\mathbf{n} + w_M\mathbf{m} + w_L\mathbf{l}\), where

\[
w_N = \xi(e_N - t_N/E_N); \quad w_M = \xi(e_M - t_M/E_T); \quad w_L = \xi(e_L - t_L/E_T) \tag{2}
\]

and \(E_N, E_T\) are the elastic normal and tangential LDPM stiffnesses, respectively (see Appendix A1).

Furthermore, at the facet level, the formulation assumes a parallel coupling between the fibers and the surrounding concrete matrix. In this case, the total stress traction on each facet can be calculated as

\[
\mathbf{t}_{\text{total}} = \mathbf{t} + \mathbf{t}_f
\]

where \(\mathbf{t}_f\) is the fiber traction.
where $A_c$ is the facet area, and $P_f$ is the crack-bridging force for each fiber crossing the given LDPM facet.

As formally noted in Eq. (3), the fiber contribution, $P_f$, to the load carrying capacity is assumed to be a function of the meso-scale crack opening, $w$, and is formulated according to a precise micro-mechanical analysis of failure mechanisms affecting the fiber resistance to pull-out. These mechanisms include debonding, friction pull-out resistance, spalling, change in fiber orientation during pull-out, snubbing, and fiber rupture. The mathematical description involved is well documented in the literature [91], discussed in detail by Ref. [88], and summarized in Appendix A2.

The LDPM formulation also simulates rate effect of concrete strength and toughness at the meso-scale by including creep and Arrhenius-type behavior. Details of the mathematical description of the two mechanical phenomena and their amalgamation within the LDPM framework are reported in Refs. [46,92]. By virtue of this formulation, LDPM (and its predecessor, the CSL model [37]) has been applied successfully to the simulation of a variety of experimental data relevant to the dynamic response of regular strength concrete [46,92]. In addition, LDPM has been recently adopted for the simulation of penetration, perforation, blast, and impact of regular strength concrete and it has shown excellent predictive capabilities [93–95].

3. LDPM-F modeling of CORTUF concrete

This section presents the numerical modeling of CORTUF, an UHPC recently developed and characterized at the Geotechnical and Structures Laboratory (GSL), United States Army Engineer Research and Development Center (ERDC) [98,99].

3.1. Mix-design, basic material properties, and internal structure

CORTUF was designed both with fiber reinforcement (CORTUF-Fiber) and without fiber reinforcement (CORTUF-Plain), and its
average composition is summarized in Table 1. The adopted sand has a maximum grain size of 0.6 mm. In addition, the basic CORTUF mix is characterized by cement content, c, equal to 794 kg/m³, which can be back calculated from the measured wet density (2442.5 kg/m³) assuming 7.5% entrained air.

It is worth observing here that for the sake of constructing the LDPM meso-structure, both sand and silica flour were considered “aggregate”, and both cement and silica fume were considered “binder”. This results in water-to-binder and aggregate-to-binder ratios of w/b = 0.15 and a/b = 1.03, respectively.

Fibers embedded in the matrix were hooked steel Drajnik ZP305 with 0.55 mm diameter, cross sectional area of 0.2376 mm², 30 mm nominal length, and a reported Young’s modulus and tensile strength of 210,000 MPa, and 1100 MPa, respectively.

LDPM-F modeling requires, as a first step, to provide an approximated geometric description of concrete internal structure at the meso-scale, i.e., the scale at which the material can be viewed as a two-phase composite in which hard and stiff inclusions are embedded in a somewhat softer and weaker matrix. In addition, LDPM-F provides realistic representation of material failure if fracture processes are associated with paths, the tortuosity of which depends on the size of the major heterogeneities. This typically happens if aggregate particles and/or inclusions are strong enough to deviate cracks and make them propagate through the embedding matrix only.

Fig. 1d shows a CT-scan of an undamaged CORTUF cylindrical specimen. A voxel intensity histogram was used in the X-ray microtomography to describe the phases of the material [100]. The visible circular holes form the pore spaces and surrounding air in the matrix, the larger darker particles are the aggregates, the cement hydrates are the smaller lighter particles, and the bright white inclusions are unhydrated particles. A particulate internal structure is clearly visible and strongly motivates the adoption of LDPM-F for CORTUF modeling. Furthermore, Fig. 1e shows a post-mortem CT-scan of the same CORTUF specimen subjected to splitting test. The crack path is affected by the presence of the major inclusions, which in most cases, are undamaged (see white arrows in Fig. 1e) and determined the location and propagation path of the visible cracks.

3.2. Coarse-grained approximation

In the process of approximating the internal structure of concrete, one of the most important parameters is the maximum aggregate size, dₐ, which for CORTUF is equal to 0.6 mm. The minimum aggregate size included in LDPM simulations is typically assumed to be d₀ = dₐ/2, so if the particle-size distribution is governed by a Fuller curve with exponent n₀ = 0.5, the percentage of simulated aggregate is about 30%. This ensures, in most cases, a good resolution of concrete internal structure and a reasonable representation of meso-scale potential crack paths. However, particle sizes ranging from 0.3 mm to 0.6 mm lead to a very large number of particles for typical numerical specimens. For example, for a 50.8 mm (2 in.) diameter and 101.6 mm (4 in.) height cylinder

Table 1

<table>
<thead>
<tr>
<th>Material</th>
<th>Proportion by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>1</td>
</tr>
<tr>
<td>Sand</td>
<td>0.967</td>
</tr>
<tr>
<td>Silica flour</td>
<td>0.277</td>
</tr>
<tr>
<td>Silica fume</td>
<td>0.389</td>
</tr>
<tr>
<td>Superplasticizer</td>
<td>0.0171</td>
</tr>
<tr>
<td>Water (tap)</td>
<td>0.208</td>
</tr>
<tr>
<td>Steel fibers (CORTUF-Fiber)</td>
<td>0.31</td>
</tr>
</tbody>
</table>

used for the CORTUF characterization in compression, one would have about 1 million particles; for the CORTUF slabs that were 304.8 mm × 304.8 mm × 50.8 mm (12 in. × 12 in. × 2 in.) and tested under projectile impact at ERDC, the particle number would be more than 20 million.

To reduce the computational cost of the simulation, a coarse-graining (CG) technique exploiting a force matching approach was adopted [101]. This coarse-graining technique is based on the following observations: (1) particle size does not influence the macroscopic LDPM response as long as the macroscopic behavior remains strain-hardening and does not feature damage localization; (2) particle size does influence the macroscopic LDPM response in the case of macroscopic strain-softening and damage localization; (3) the main parameter governing LDPM strain-softening behavior is the material characteristic length, defined as

\[ l_t = \frac{E_0 G_t}{\sigma_t^2} \]

where \( E_0 \) = normal modulus, \( G_t \) = meso-scale tensile fracture energy, and \( \sigma_t \) = meso-scale tensile strength. For a coarse-graining factor \( \kappa = \frac{d_{\text{coarse}}}{d_0} = \frac{d_{\text{coarse}}}{d_0} \) and with the same Fuller coefficient for both the coarse and the fine system, the macroscopic coarse response can be shown to be a good approximation of the fine response if the coarse characteristic length is calculated as

\[ l_t^{\text{coarse}} = \frac{l_t^{\text{fine}}}{\kappa} \]

(4)

where \( l_t \) = interparticle distance, which is locally variable according to the grain size distribution, and the overbar represents the mean values of the interparticle distance distributions. It must be observed that in general, \( l_t^{\text{coarse}} / \kappa \neq l_t \), because the LDPM system is not composed by close-packed spheres and, consequently, the interparticle distance distribution does not coincide with the particle size distribution. In this paper, a maximum aggregate size equal to 4 mm and a Fuller coefficient equal to 0.5 were adopted for the coarse system. Fig. 1f shows a 20 mm specimen with visible particles for maximum aggregate sizes of 4 mm (left) and 0.6 mm (right); Fig. 1g shows the associated Cumulative Distribution Function (CDF) of the inter-particle distance. In this case, \( l_t^{\text{coarse}} / \kappa = 5.5 \) is obtained.

The accuracy and effectiveness of the proposed CG technique is investigated in Fig. 1h and i, which compare the average response of three fine (dashed curve) and three coarse (solid curve) LDPM specimens under unconfined compression tests and splitting tensile tests, respectively. The simulated specimens were 20 mm side cubes comprised of an average of 51,000 and 450 particles for the fine and coarse system, respectively. Simulations were performed on the Diamond system at the ERDC Department of Defense Supercomputing Resource Center, having 2.8 GHz Intel Xeon X5560 Nehalem-EP processors on its login and compute nodes, with 2 processors per node, each with 4 cores, for a total of 8 cores per node. The simulations utilized 1 node and were characterized by a run time of 10 h and 20 min (compression) and 5 h and 10 min (splitting) for each run of the fine and coarse systems, respectively. As can be seen from the results obtained, the coarse approximation does provide very accurate results with a significant saving in terms of computational time; the difference in the peak response is about 7% for the compression test and 6% for the splitting test. The observation must be made that the analyzed examples are characterized by a damage zone whose size is larger than the coarse maximum aggregate size. The same level of accuracy cannot be expected for cases in which this condition is not satisfied.

3.3. Fiber reinforcing

Fiber reinforcing is always used in UHPC to increase toughness (energy absorption capability) and ductility. Within LDPM, fibers
are explicitly modeled by generating spatial distributions mimicking the ones found in real specimens. From a geometric point of view the physical fibers can be described using few parameters, in the case of straight fibers: volume fraction, \( V_f \), length, \( L_f \), and equivalent diameter, \( d_e \). Each fiber is modeled using a sequence of one or more segments linked together. Single segments are sufficient for generating straight fibers. Multiple segments are necessary for generating tortuous fibers.

These geometric parameters are used for generating random fibers inside a control volume. For cast fiber-reinforced concrete specimens, the control volume is assumed to be equal to the volume of the concrete part. In this case, fiber location and orientation are affected in the vicinity of the boundary. On the contrary, for machined or cored specimens, the control volume is assumed to be larger than the actual specimen so that fibers intersecting the external surface of the specimens are treated as cut. The portion of a cut fiber inside the specimen is shorter than the length of original fiber, and this affects the mechanical characteristics of the fiber–concrete interaction near the specimen boundaries.

The following algorithm is used for generating random fibers whose number inside a given control volume \( V \) can be computed as \( N = 4V/V/(L_f \pi d_e^2) \). The first scenario details the case of uniformly distributed fibers. For each fiber, a random point with uniform probability in space inside the control volume is selected as the initial point of the fiber. An initial random direction is then computed by (1) generating, in a unit cubic volume, a series of additional random points with a uniform probability in space, (2) rejecting all points outside a unit sphere, and (3) accepting the first point occurring inside the unit sphere. The vector connecting the fiber origin to this point inside the unit sphere identifies an unbiased spatial direction. The computed direction is used to initialize the fiber. If the fiber consists of multiple segments to simulate fiber tortuosity, then the direction of each additional segment is modified using the following logic.

A random direction, \( \mathbf{w} \), is computed as done previously, followed by the normal \( \mathbf{n} \) to the current fiber direction \( \mathbf{u}_1 \) as \( \mathbf{n} = \mathbf{m}/|\mathbf{m}| \), where \( \mathbf{m} = \mathbf{w} - (\mathbf{w} \cdot \mathbf{u}_1)\mathbf{u}_1 \). A proportional component is then added to the tortuosity factor \( t \) by \( \mathbf{v}_2 = \mathbf{u}_1 + t \mathbf{n} \), and the new fiber direction \( \mathbf{u}_2 \) is normalized using \( \mathbf{u}_2 = \mathbf{v}_2/|\mathbf{v}_2| \). For \( t = 0 \), the fiber continues straight in the initial direction. If the fiber exits from the control volume \( V \), the fiber is discarded.

In some cases, fiber orientation is not random but has a bias towards a preferential direction. This bias is obtained by a scaling factor, \( s \), for a stated preferential direction. For \( s = 1 \), there is no preferential alignment, while a generic value of \( s \) makes alignment \( s \) times more likely for the preferential fiber orientation. The most appropriate scaling factor must be chosen through comparisons between numerical and experimental specimens. The random fiber generation algorithm is adjusted to include preferentially oriented fibers after the spatially random direction is computed. The dot product \( \mathbf{c} = \mathbf{n} \cdot \mathbf{d} \) of random \( \mathbf{n} \) and preferential orientation \( \mathbf{d} \) is computed. The random direction is then extended along the preferential direction and normalized to compute the random fiber direction by \( \mathbf{v} = \mathbf{n} + (s - 1)\mathbf{d} \) and \( \mathbf{n} = \mathbf{v}/|\mathbf{v}| \). Figs. 4a and c show generated fiber distributions without and with preferential orientation.

In the spirit of the discrete multi-scale physical character of LDPM, the occurrences of fiber-facet intersections are determined by actually computing the locations where fibers cross inter-cell facets. This computation can require significant resources for large models in terms of both time and memory. For this reason, an efficient bin-sorting algorithm was developed for computing these intersections. For each intersection, the lengths of the fiber on both sides of the facet and the angle at which the fiber intersects the facet are also computed. These parameters are saved in the facet data structure and used during the simulation for computing the fiber effect.

The modeling of fiber reinforcement in CORTUF is then completed with the simulation of fiber–concrete interaction, which requires the formulation of a local force versus slippage constitutive equation simulating fine-scale failure phenomena occurring at the interface between fibers and embedding concrete matrix. The LDPM-F formulation [88,89] of such a law, adopted in this study and summarized in Appendix A2, includes the description of the following failure mechanisms, (1) fiber/matrix debonding, (2) fiber frictional pull-out, (3) local matrix failure due to stress concentration at the point of exit of fibers from the matrix (micro-spalling), (4) pull-out strength increase due to additional frictional effects caused by change of direction of fiber during cracking (snubbing), (5) fiber rupture, (6) fiber strength reduction due to localized plastic deformations, and (7) fiber yielding due to local fiber bending.

In addition to these failure mechanisms, there is evidence for CORTUF concrete that splitting failure, i.e., a sudden and dynamic propagation of a crack in a plane containing the fiber axis, might contribute significantly to the overall tensile resistance in tension of the material. Let us consider a straight segment of a fiber subject to tension. At the interface between the fiber and the surrounding concrete, shear stresses develop, which in turn lead to inclined principal stress directions. When the maximum principal stress exceeds the tensile strength, radial cracks initiate, and the concrete matrix surrounding the fiber is forced to transfer load through an inclined compression field, see Fig. 1c. The component of this inclined field acting orthogonally to the axis of the fiber results in an expansive pressure on the surrounding concrete of magnitude \( p = P \tan \alpha (\pi d F) \). The angle \( \alpha \) depends on both the geometric characteristics of the fiber and the remote stress state. From this pressure, the maximum circumferential

![Fig. 2. Effect of high strain rate on compression test of UHPC](image-url)
tensile stress acting at the fiber/matrix interface (see Fig. 1c) can be computed approximately as

\[
\sigma_{\text{max}}^q = \frac{p}{4c_f^2/d_f^2 + 1}
\]

where \(c_f\) is the distance between the fiber axis and the closest external surface of the specimen.

By introducing the expression for \(p\) into Eq. (5), setting the failure condition, \(\sigma_{\text{max}}^q = \sigma_t^c\), and solving for the fiber force, one can calculate the splitting force

\[
P_{\text{split}}^f = \frac{\pi d_f L \sigma_t^c}{\tan \alpha} \left( \frac{4c_f^2/d_f^2 - 1}{4c_f^2/d_f^2 + 1} \right) = \frac{\pi d_f L \sigma_t^c}{\tan \alpha} \text{ for } c_f/d_f \gg 1
\]

where \(\sigma_t^c\) can be assumed to coincide with the tensile strength of the matrix or the composite, depending on fiber concentration and level of fiber interaction. At this point, a few observations are in order. (1) The splitting mechanism highlighted below is typically observed in reinforced concrete members [102] when concrete rebars are not provided with proper confinement through either enough concrete cover or transverse reinforcement. It is worth pointing out that, in regular concrete, rebars have diameter sizes of the same order of the aggregate particles; the same situation arises in CORTUF concrete where fiber diameters have similar characteristic sizes of material heterogeneities. Based on this analogy, it seems logical to observe in CORTUF failure mechanisms that are similar to the ones of reinforced concrete and that are not observed in standard fiber-reinforced concrete. (2) Splitting failure is expected to be negligible in the case of fiber reinforcement with low stiffness and high Poisson’s ratio compared to the one of concrete (e.g., for Polyvinyl Alcohol, PVA, fibers), because such properties prevent the build-up of a significant expansive pressure. (3) If bond resistance is limited, bond failure occurs before the inclined cracks can develop. As such, straight fibers are less prone to induce splitting failure than hooked and bent fibers.

3.4. Rate effect

Due to the specific curing protocol used to manufacture CORTUF, it can be assumed confidently that capillary and absorbed water content is very small and actually close to none. For this reason, intrinsic phenomena such creep and moisture effect can be neglected in the simulations. Arguably one could include the Arrhenius-type effect but this would require identification of the relevant parameters, which is possible only through the analysis of rate-dependent experimental data that is not available at the moment for CORTUF. Consequently, a choice was made to neglect any intrinsic phenomena in the LDPM-F formulation for CORTUF.

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Fig. 3. Tests results for the parametric identification of LDPM-F for a) 3-point bending, b) uniaxial strain and triaxial compression, c) single fiber pull-out.

Fig. 4. a) Random fiber orientation, b) CT scan of fiber orientation, c) preferential fiber orientation, d) validation test results for 3-point bending, e) failure mode for 3-point bending specimen on CORTUF-Plain, f) failure mode for 3-point bending specimen on CORTUF-Fiber.
However, the model is still able to capture the rate-effect associated with apparent mechanisms. Fig. 2a shows the DIF obtained by simulating a cylindrical (50.8 mm × 101.6 mm) CORTUF specimen fixed at the base and subjected, at the top, to various load rates corresponding to nominal strain rates (velocity of the top side of the specimen divided by the specimen length) from 0.001 s⁻¹ to 10 s⁻¹. As one can see the response shows an increase of the macroscopic compressive strength up to 1.24 times the quasi-static value. This increase results solely from inertia effects and change in crack patterns (Fig. 2b–d). Finally, it is worth noting that the adopted formulation cannot capture apparent mechanisms occurring at a length scale smaller than the adopted minimum LDPM particle size (2 mm in this study). However, introduction in the constitutive equations of these fine scale effects would be only a futile academic exercise without relevant and appropriate experimental data needed to calibrate and validate the resulting more complicated model.

3.5. Model parameter identification

The LDPM-F constitutive equations at the facet level depend on a number of model parameters (see Appendix A2) that need to be identified by fitting experimental data. Extensive discussion of LDPM and LDPM-F calibration procedures is reported in Refs. [86,89]. In this study, first, the elastic parameters were calibrated by matching the macroscopic elastic modulus obtained from the average slope of the CORTUF-Plain and CORTUF-Fiber unconfined compression curves (51.591 MPa) and Poisson’s ratio as determined by Ref. [103] having a value of 0.115. By using the macro-to-meso formulas reported in Ref. [85], one obtains the normal elastic modulus, \( E_N = 67,000 \) MPa, and the shear-normal coupling parameter, \( \alpha = 0.484 \). Experimental data from both CORTUF-Plain and CORTUF-Fiber were used, because the LDPM-F formulation neglects the effect of fiber reinforcement on the elastic behavior. This is a reasonable approximation for low-fiber-volume fractions as the ones typically used in concrete (<10%). Next, identification of the other concrete static parameters was performed by matching the experimental results of CORTUF-Plain in four different tests, namely, uniaxial strain (UX), uniaxial unconfined compression (UC), triaxial compression (TXC) with 300 MPa confinement, and 3-point bending (3PBT) on notched specimens. Finally, the LDPM-F parameters related to concrete–fiber interaction were identified by the best-fit of single pull-out tests relevant to two fiber embedment lengths, 6.35 mm (0.25 in.) and 12.7 mm (0.5 in.). Table 2 summarizes the parameters specific to each test as well as the resulting identified values. Model parameters whose calibration require experimental data not available for CORTUF were assumed on the basis of standard values available in the literature [86] and reported in Appendix A. In order to account for the variation that concrete exhibits in experiments, the meso-structure for each simulated test was generated three times, and simulations were performed on each mesh generation. Similarly, fibers in the fiber-reinforced CORTUF specimens were generated three times to reproduce the effect of fiber distribution. The average of the numerical results is plotted for the numerical data discussed hereinafter. Experimental data are compared with the numerical ones either with average curves, circles points in the figures, or with curve ranges, shaded gray areas in the figures, identified between the minimum and maximum envelopes of the experimental curves. In addition, unless otherwise specified, the comparison is done on the basis of simulations performed on the coarse-grained model. The numerical simulations were performed with the Modeling and Analysis of the Response of Structures (MARS) software [104].

### Table 2: Tests performed for calibration and values of LDPM parameters in simulations.

<table>
<thead>
<tr>
<th>Test performed</th>
<th>LDPM parameter</th>
<th>UHPC</th>
<th>UHPC-L</th>
<th>UHPC-U</th>
<th>RSC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniaxial strain</strong></td>
<td>1. Compressive strength</td>
<td>500</td>
<td>425</td>
<td>575</td>
<td>150</td>
</tr>
<tr>
<td>2. Initial hardening modulus ratio</td>
<td></td>
<td>0.36</td>
<td>0.306</td>
<td>0.414</td>
<td>0.4</td>
</tr>
<tr>
<td>3. Densification ratio</td>
<td></td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>1</td>
</tr>
<tr>
<td>4. Transitional strain ratio</td>
<td></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td><strong>Uniaxial unconfined compression</strong></td>
<td>5. Shear strength ratio</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>2.7</td>
</tr>
<tr>
<td>6. Initial friction</td>
<td></td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>7. Softening exponent</td>
<td></td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Triaxial compression</strong></td>
<td>8. Deviatoric strain threshold ratio</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>9. Deviatoric damage parameter</td>
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<td>1</td>
<td>1</td>
<td>5</td>
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<tr>
<td>10. Asymptotic friction</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3-Point test (Notch)</td>
<td>11. Transitional stress</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>600</td>
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<tr>
<td></td>
<td>12. Tensile strength</td>
<td>4</td>
<td>3.4</td>
<td>4.6</td>
<td>4.03</td>
</tr>
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<td></td>
<td>13. Tensile characteristic length</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>120</td>
</tr>
<tr>
<td>Single fiber pull-out</td>
<td>14. Bond strength</td>
<td>11.5</td>
<td>11.5</td>
<td>11.5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>15. Coarse aggregate</td>
<td>4 mm</td>
<td>4 mm</td>
<td>4 mm</td>
<td>8 mm</td>
</tr>
</tbody>
</table>

Tensile strength and toughness parameters of LDPM were identified by simulating 3PBTs, which were performed on specimens that had a 50.8 mm (2 in.) width, a 25.4 mm (1 in.) depth, a 203.2 mm (8 in.) span, and a notch dimension of 15.24 mm (0.6 in.) depth and 5.08 mm (0.2 in.) wide. Load versus CMOD (Crank Mouth Opening Displacement) curve comparison for the experimental and numerical results are shown in Fig. 3a.

Fig. 3b shows the best fitting obtained for the Uniaxial Strain (UX) test, characterized by zero lateral expansion, as well as Triaxial Compression (TXC) tests at 0 MPa (Unconfined Compression, UC) and 300 MPa lateral confinement (TXC300), respectively. These tests were used to identify the compression-related LDPM parameters. In Fig. 3b, the stress versus strain curves are offset by 0.05 for clarity of representation, and the unloading branches of the UX and TXC300 are included. Compression tests were all conducted on cored cylindrical specimens having lengths of 101.6 mm (4 in.) and diameters of 50.8 mm (2 in.) and were loaded through loading platens treated with friction reducing materials in order to minimize the effect of friction between the platens and the specimen ends. Additional details of the experimental procedure can be found in Refs. [98,99]. As can be seen in Fig. 3b, LDPM-F can accurately reproduce the behaviors of CORTUF at various levels of confinement, i.e., very brittle strain-softening behavior at no confinement and ductile and strain-hardening behavior at high confinement. The model also nicely captures CORTUF unloading behavior.

Finally, the calibration of LDPM-F requires the identification of the material parameters governing the fiber–matrix interaction model. The basic test used for this task is the single fiber pull-out (SPO) test. When a fiber is pulled out of concrete, the initial resistance is due to both debonding fracture energy as well as frictional resistance [105]. Once the fiber debonds completely from the matrix, the resistance shifts to purely frictional, which can be either slip-hardening, increasing pull-out force for increasing slippage, or slip-softening, decreasing pull-out force for increasing slippage. This feature of the pull-out behavior can be simulated through the
A decision was made to adopt slip-hardening, whereas the longer embedment length displays ciding with the and 6.35 mm (0.5 in. and 0.25 in.), and direction of pulling coinciding with the fiber axis. The shorter embedment length shows slip-hardening, whereas the longer embedment length displays more of a slip-softening response, and they can only be fitted with two distinct values of the parameter $\beta$ (see dashed lines in Fig. 3c). A decision was made to adopt $\beta = 0$ (see solid lines in Fig. 3c) that provides an overestimate of the pull-out capacity of the longer embedment length and an underestimation of the shorter. The inability of the model to capture the differences in behavior for different embedment lengths is due to the presence of hooks at fiber ends that are not accounted for explicitly in the analytical model [91].

However, this limitation is not crucial for the present study, since only a small portion of the frictional pull-out resistance affects the behavior of the overall composite, which is actually governed by other failure mechanisms. One of these mechanisms is the splitting failure discussed earlier in this paper (see Eq. (6)). In absence of specific experimental data, the following assumptions are employed, $\sigma_t = \sigma_1 = 4$ MPa, $L_{f} = L_{f}/2 = 15$ mm, and $\alpha = \pi/6$.

This leads to an average splitting resistance of $f_{\text{split}} = 179$ N. Note that the value $\alpha = \pi/6$ is motivated by the likely occurrence of compressive stresses parallel to the fiber axis induced by the fiber hooks and leading to a reduction of the inclined crack orientation, which would be $\pi/4$ in the case of pure tangential stresses (note that similar behavior is observed for shear cracks in prestressed concrete members).

Other failure mechanisms include fiber rupture with inclined pulling, micro-spalling, and snubbing, whose governing parameters (reported in Appendix A2) were assumed on the basis of Ref. [88].

3.6. Model validation and discussion

This section presents the validation of the formulated model. Validation simulations are necessary to prove that LDPM-F serves its intended purpose of predicting the mechanical responses of CORTUF-Plain and CORTUF-Fiber. All prediction simulations were executed with no further change to the previously calibrated LDPM-F parameters, and their results are compared to experimental data that were not utilized during the parameter identification.

3.7. Fracture tests

The same notched 3PBT used for the fracture characterization of CORTUF-Plain was also used for CORTUF-Fiber. The experiments were intended to be performed with a random distribution of fibers (Fig. 4a) and with a fiber volume fraction equal to $V_f = 3\%$ correspoding to the fiber-to-binder ratio in mass equal to 0.31 reported earlier in Table 1.

Numerical results of the corresponding numerical simulations showed lower peaks compared to the experimental data as shown in Fig. 4d. However, computer tomography (CT) scans [100] revealed $V_f$ to be between 3.5 and 4% in the vicinity of the notch tip as opposed to the nominal 3%. In addition, the scans provided insight into the fiber placement and distribution; fibers were not distributed randomly (Fig. 4a) as intended but rather had a preferential orientation in the direction orthogonal to the notch (Fig. 4b and c).

Fig. 4d shows the load versus mid-span-deflection curve for CORTUF-Fiber with random (thin solid curves) and preferential (thick solid curves) fiber orientation for $V_f$ equal to 3, 4, and 5%. Simulations with random fiber orientation are consistently lower than the experimental results with the curve relevant to the highest volume fraction barely reaching the experimental lower bound. On the contrary and in agreement with the CT scans, the predictions with preferential fiber orientation coincides with the experimental results very well. Comparison of the 3PBT CORTUF-Plain result (see Fig. 3a) with the 3PBT CORTUF-Fiber result (see Fig. 4d) demonstrates that the effect of fibers leads to an increase of the carrying capacity of more than four times, from about 0.9 kN to 3.8 kN (for 4% fiber volume fraction). Furthermore, the capability of the material to dissipate energy increases by orders of magnitude. Numerical fracture energy values computed through the work-of-fracture concept [71] give estimates of 30 N/m and 9800 N/m for CORTUF-Plain and CORTUF-Fiber ($V_f = 4\%$), respectively, at 40% loss of load carrying capacity.

Figs. 4e and f report the crack distributions at 0.1 mm and 5.9 mm deflection for the CORTUF-Plain and CORTUF-Fiber, respectively. Unlike the unreinforced specimen, the fiber addition creates a more distributed crack pattern with a significant increase of the fracture process zone (FPZ) size.

The numerical response of CORTUF-Fiber and the associated failure mechanisms are further investigated in Fig. 5a. In this figure, the results of the numerical simulations for $V_f = 4\%$ obtained by predicting both fiber rupture (case splitting $P_1=V_f P_2$) and prevent bond splitting only (curves $P_2$ and $P_3$) are compared with the response accounting for all failure mechanisms (curves $P_1$ and $R_1$). The prefix “$P$” indicates preferential fiber orientation whereas “$R$” indicates random fiber orientation. Additionally, the CORTUF-Plain results are included for comparison as Plain. As one can see, the dominating failure mechanism is bond splitting. When this failure mechanism is prevented, the load carrying capacity more than doubles, i.e., at 3 mm deflection, the ratio between the load curves $P_2/P_1$, and $R_2/R_1$ is 2.4 and 2.3, respectively. On the contrary, fiber rupture does not have a significant role as demonstrated by the comparison between $P_2$ and $P_3$ as well as $R_2$ and $R_3$. Moreover, results show a slightly more important effect of fiber rupture for the random orientation case. This is due to the fact that, in this case, there is a higher likelihood for fibers to bridge cracks at an angle leading to fiber bending with associated localized damage at the bent location [91,106–109].

The conclusion that fiber bond splitting might be the dominant mechanism is also confirmed by performed CT-scan analyses, which did not show any significant occurrence of fiber rupture or even necking and, at the same time, provides evidence of possible cracks propagating along the fiber axis (see Fig. 6a).

Furthermore, post-mortem evaluation of tested specimens showed the presence of undamaged fibers (with even intact hooks and bends) across the crack surface (see Fig. 6b). This is not consistent with fiber rupture, leading to the separation of the fibers in two pieces, or pull-out, leading to plastic deformation and straightening of the fiber hooks, but it is completely consistent with the bond splitting mechanism hypothesized in this paper.

For the case with fiber preferential orientation, the onset of bond splitting failure occurs for a load level of about 3.5 kN as shown in Fig. 5a by the point at which the $P_1$ and $P_3$ curves start to diverge. To further investigate this phenomenon, crack opening distribution with cumulative distribution function plots are shown in Fig. 5b and c for a centerline displacement of 1.2 mm (dotted line in Fig. 5a), which is about 75% of the maximum stress. Fig. 5b shows the comparison of the random fiber distributions, while the results of fibers with preferential orientation are shown in Fig. 5c. The relative crack distribution of the plain specimen at 75% of its maximum stress is also included in the plots. At the onset of failure, $P_1$ and $R_1$ exhibit slightly higher crack distributions than $P_2$, $P_3$, and $R_2$, $R_3$, respectively. As the crack openings become greater than about 100 $\mu$m for the random distribution, there is a change in response coefficient $\beta$ (see Ref. [88] and Appendix A) that is $>0$ in the case of slip-hardening and $<0$ in the case of slip-softening.
as the coupled $R_2$ and $R_3$ are able to bridge more of the larger cracks. Lastly, comparisons between the plain and the reinforced specimens clearly show the crack-bridging effect of the fibers. For the plain specimens, the cracks governing failure are magnitudes less than the reinforced specimens. The inability of cracks to propagate beyond a few $\mu$m results in localized failure.

### 3.8. Tensile strength tests

The tensile strength of the CORTUF-fiber was investigated through four-point bending tests and splitting (Brazilian) tests. Three different specimen sizes were used for the 4-point (4pt) tests—Size A, Size B and Size C (see Fig. 7a). The width for all specimens was 102 mm, and the other dimensions in units of mm are as follows: Size A ($l_1 = 101$, $l_2 = 305$, $l_3 = 356$, $h = 25$), Size B ($l_1 = 101$, $l_2 = 305$, $l_3 = 356$, $h = 102$), and Size C ($l_1 = 304$, $l_2 = 914$, $l_3 = 1016$, $h = 102$). In the 4pt test, a concrete block was supported on two supports, and a load was applied through a loading plate at the top of the specimen. To imitate the loading plate in the experimental setup, an additional elastic plate was used to apply load on top the specimen in LDPM. Unlike the cylindrical and largest 4pt specimens that were cast, ERDC reported that the smaller size 4pt specimens (A and B) to be cut, and this feature was also reproduced in the LDPM simulations.

Simulations with both random and preferential orientation of the fibers were performed and, similarly to the 3PBT, results show that the fibers were preferentially oriented in the specimen in the longitudinal direction. Numerical and experimental nominal stress, $\sigma_{N} = 3fl_{2} - l_{1}h^{2}$, versus nominal strain, $\varepsilon_{N} = 12h_{2}l_{2}(3l_{2}^{2} - 4l_{1}l_{2} - l_{1}^{2})$, curves are reported in Fig. 7b–d, for the specimen sizes A, B, and C, respectively. For the 4% fiber volume fraction, the peak nominal stress was 55.8, 51.7, and 51.7 MPa for the three different sizes that corresponds to 24.8% and 23% of the compressive strength. These values are in very good agreement with the respective experimental values of as shown in Fig. 7b–d, but it must be noted that both experimental and numerical data provide a significant overestimation of the actual tensile strength because, due to the fiber crack bridging effect, the specimens responses show a pronounced nonlinear behavior prior to the peak suggesting a stable crack propagation after crack initiation and before reaching the load carrying capacity. The numerical results and the experimental data are also compared in Fig. 7e as far as the crack distribution is concerned, showing the excellent ability of LDPM in reproducing realistic crack patterns.

The cylindrical specimens used for the splitting tests were characterized by a radius of 51 mm (2 in.) and length of 204 mm (8 in.). The test was performed according to the ASTM standard, and a compressive load was applied along the specimen’s length with rectangular rods that prevented crushing of the cylinder underneath the applied load. The rod had dimensions of 10 mm width, 226 mm length, and 5 mm height. In a splitting test, the maximum stress occurs at the center of the specimen in the direction orthogonal to the applied load and decreases closer to the supports until eventually stresses at the support become negative (compression). This leads to a longitudinal crack that splits the cylinder into two pieces as Fig. 7f demonstrates by comparing postmortem images of both an experiment and a simulation.

Peak stress numerical results obtained for the CORTUF-Fiber specimens containing $V_f$ values of 3, 4 and 5 were 10.8 MPa, 12.3 MPa, and 13.8 MPa respectively, and the peak stress for the CORTUF-Plain specimen was 7.2 MPa. The average experimental splitting stress for CORTUF-Fiber was 25.3 MPa, and for CORTUF-Plain was 10.8 MPa. The discrepancy in these values, especially for CORTUF-Fiber, can be explained by the fact that a compressive stress field exists in the direction parallel to the crack path, and consequently, transverse to the crack bridging fiber during splitting failure. Such transverse confinement can significantly influence the pull-out resistance by preventing certain failure mechanisms (e.g., splitting and spalling) and by increasing the frictional resistance. This phenomenon is not currently handled in LDPM-F, because the fiber–concrete interaction model does not depend on the confining stress acting transversely to the fiber axis.

![Fig. 5. 3-Point bending test results for a) stress–strain curve failure mechanisms; crack distribution function for b) random fiber orientation, c) preferential fiber orientation.](image-url)
3.9. Uniaxial, triaxial, and hydrostatic compressive tests

Fig. 8 shows comparisons between experimental data and numerical simulation results with reference to triaxial tests on CORTUF-Plain and CORTUF-Fiber that were not used in the calibration phase. For completeness, both experimental and numerical data, previously simulated for the purpose of parameter identification are reported in light gray.

Figs. 8a and d report CORTUF-plain and CORTUF-fiber responses, respectively, under uniaxial strain compression and hydrostatic compression. The results are plotted in terms of volumetric stress versus volumetric strain, and consequently they feature the same elastic portion of the curve represented by the volumetric elastic modulus, $E_V = E/(1 - 2\nu) = 67000$ MPa. In the inelastic regimes, the two curves are distinctly different. Results show that LDPM-F can capture such different behavior, unlike most other constitutive equations available in the literature. LDPM-F can also capture the behavior for unloading very well. It is worth noting that the difficulties of most models to capture the difference between the volumetric stress versus volumetric strain curves...
obtained under uniaxial strain and hydrostatic compression are associated with the interpretation that these curves are an "equation of state". As many other experiments have already shown, this interpretation cannot be used in the case of concrete, and frictional materials in general.

For the uniaxial strain tests, the numerical versus experimental comparison is further investigated in Fig. 8b and e with plots of principal stress difference versus volumetric stress. Once again, LDPM-F is able to reasonably simulate the CORTUF response during both the loading and unloading phases of the test.

Simulation of the stress versus strain curves from the TXC tests on CORTUF-Plain with varying transverse confinement (10, 20, 50, 100, 200 MPa) not used in the parameter identification phase is shown in Fig. 8c where the curves are offset by 0.05 for clarity of representation. Similarly, Fig. 8e presents the fiber comparison with the CORTUF-Fiber TXC test results with varying transverse confinements (0, 10, 20, 50, 100, 200, 300 MPa). As one can see, the experimental data suggests a very small effect of the fibers on the overall compressive response of CORTUF. This is consistent with the LDPM-F assumption that fibers are engaged only for cracks displaying tensile normal opening and that they do not contribute significantly to the resistance of the composites under combined meso-scale compression and shearing.

Furthermore, CORTUF shows a significant increase of strength upon confinement. For example, for a transverse confinement of 100 MPa, the strength for both CORTUF-plain and CORTUF-fiber is in excess of 550 MPa, corresponding to over two times the unconfined compressive strength. LDPM-F simulates the strength increase for all levels of confinement considered with excellent accuracy. In addition to strength, the presence of confinement significantly affects the ductility of the material. The experimental data shows a transition from a brittle strain-softening response to a ductile strain-hardening response for increasing confinement, and the transition seems to occur at a level of confinement between 100 and 200 MPa. LDPM-F simulates the brittle-to-ductile transition but predicts the transition confinement to be at about 100 MPa. In addition, for 20 MPa and 50 MPa confining pressure, the numerical responses seem to overestimate the material ductility, even though this cannot be known with certainty since the experimental data does not have the post peak portion of the curve.

The transition from a brittle-to-ductile behavior in compression is also clearly seen in Fig. 9a–d that display crack patterns at failure with varying confinement. For increasing confinement, the failure evolves from a brittle split crack, to one that resembles the occurrence of shear bands, and then a complete, uniformly distributed crushing that leads to the observed increased ductility.

4. Numerical simulation of projectile penetration into CORTUF slabs

This section discusses the use of LDPM for the simulation of penetration and perforation experiments.

In the recent past, various researchers have attempted this type of simulations by using other computational technologies. The Finite Element Method with element deletion [110] is the simplest way to simulate arbitrary propagating cracks and it simulates cracks by removing elements from the mesh when a certain criterion is met. This method requires a very fine mesh to provide reasonably accurate results and it cannot be used to simulate the effect of secondary fragments since most of the failed material disappears from the simulation.

Meshfree methods, such as the Smoothed Particle Hydrodynamics (SPH) method [111,112], have gained some popularity in the simulation of penetration mechanics for their ability of allowing very large deformations. Successful examples of application of SPH to the simulation of perforation through concrete can be found in Refs. [113–115]. However, the main problem of classical mesh-free methods is the underlying assumption that concrete can be treated as an isotropic continuum even during failure and fragmentation. Consequently, the transition from continuum to discrete is handled through numerical artifacts as opposed to an actual loss of continuity of the governing mathematical formulation.

A solution to this problem is to describe displacement discontinuities explicitly. Meshless methods enhanced with the description of displacement discontinuities were used successfully in Refs. [116,117] to simulate dynamic failure of concrete. However, this method still neglects the heterogeneous character of concrete and requires, unlike LDPM, additional degrees of freedom to represent the evolving discontinuities making the computational cost to increase significantly during fracture and fragmentation compared to the unfractured state.

The simulated perforation experiments were performed at ERDC on three square 304.8 mm (12 in.) CORTUF-Fiber prismatic slabs. The slabs, labeled A, B, and C, had widths 50.8 mm (2.0 in.), 63.5 mm (2.5 in.), and 76.2 mm (3.0 in.), respectively. A fragment simulating penetrator (FSP) projectile was used in the experiments. Although one of each specimen size was tested experimentally, LDPM allowed for numerous simulations in a cost-effective and time-efficient manner for a wide range in velocities below, at, and above the ballistic limit of each panel, as well as various fiber-volume fractions (3, 4, and 5%).

The projectile was modeled as a plastic high-strength steel with an elastic modulus of 200 GPa. The classical Flanagan Belytschko Single Integration Point finite element algorithm with

![Fig. 9. Triaxial compression crack pattern comparison for CORTUF-Plain for a) 0 MPa, b) 10 MPa, c) 50 MPa, and d) 200 MPa confinements.](image-url)
hour-glass stabilization was used for its speed and accuracy with a hexahedral mesh. Additionally, there was no constraint on the FSP for either translations or rotations. Fig. 10a shows the side, front, top, and back views of the FSP meshing. For the large contact forces exerted on the projectile, greater deformation was observed for the FSP in the simulation compared to experimental results. However, due to the caliber having a relatively flat nose as seen in Fig. 10a side and top views, the deformations were not excessive enough to arrest the simulation. A penalty [104] contact algorithm was adopted between the FSP and the panel, which utilized the relative displacement of the two nodes lists, i.e., FSP node list and panel node list. The dimensionless nodes for the panel and bullet were assumed to be spherical with 1 mm and 2 mm thicknesses respectively for the contact, which occurs when the two spherical external surfaces are in contact. Contact conditions between the nodes of the same list were omitted in the model.

The schematic of the test setup portrayed the panel in a holder held only at the outer edges. With no definite knowledge about the exact boundary conditions of the panel during the experimentation, all surface nodes for the panel width zone were held fixed with no possibility of displacement or rotation, and further investigation of the effects of varied boundary conditions were performed. Results of this boundary investigation are discussed later in the paper.

The average ballistic limit for the projectile impacting three different unreinforced specimens of Type A, the smallest panel, was 1706 m/s. When a target is impacted, a compressive shock wave passes through the specimen and is reflected at the rear face as a tensile wave. Ultimately, expulsion of the projectile from the front face causes spalling. In addition, when the reflected tensile stress wave exceeds the tensile capacity of the material, scabbing occurs on the rear face [19]. Figs. 10b–d display the complete evolution of the phenomenon for an impact velocity greater than the ballistic limit. The change in velocity of the FSP with time is presented in Fig. 10b and c shows the penetration depth of the projectile with time, and Fig. 10d displays the failure patterns observed for the panel at 0.005 ms time increments in LDPM for each stage of impact for the side–panel view section. Figs. 10b and c separate three stages by vertical dotted lines. The initial decrease in velocity slope is due to the compressive behavior of the impact, the first change in velocity slope, and is the first stage; the second stage, governed by compression/shear corresponds to a tunneling effect in the panel; and finally, the rear-face scabbing coincides with the final change in velocity slope, the third stage before the curve plateaus upon projectile exit. Since the width of Panel A is only 50.8 mm (2.0 in.), the tunneling effect is restricted, and the failure observed has limited tunneling effect, as opposed to Sizes B and C.

Results depicting the change in velocity with time for impact velocities ranging from 15 m/s to 3353 m/s, below and above the ballistic limit, are shown for a CORTUF-Plain specimen A in Fig. 11a and b, respectively. For impact velocities below the ballistic limit, there was 100% attenuation of the exit velocity until the ballistic limit, as shown in Fig. 11a. Fig. 11b shows a decrease in velocity attenuation for increases in projectile impact velocity. Additionally,
the variation in energy (normalized) observed throughout the experiment for the ballistic limit of CORTUF-Plain, is shown in Fig. 11c. The kinetic energy (KE), the energy given to the system, decreases as the velocity of the bullet decreases with time since the energy of the projectile governs and is directly proportional to the square of the projectile velocity. The internal energy (IE) in the plots reference the work done by internal forces, which results in recoverable elastic energy and dissipated energy. The internal energy in the concrete (IEC) and the bullet (IEB) combine to form the total internal energy in the system. The dissipated energy in the concrete (DE) is the unrecoverable energy; it is less than the internal energy in the concrete.

Further investigation was performed on the FSP considering the effect of using elastic and rigid projectiles for the numerical simulations. Fig. 11d shows a comparison of velocity change with time for an impact velocity above the ballistic limit for plastic fsp, and rigid fsp. The results show the disparity between velocities of the elastic and rigid projectiles, and the plastic projectile that absorbs energy. The deformation of the plastic penetrator is more than that in the elastic and the rigid, which results in an additional 100 m/s residual velocity for the elastic and rigid projectiles. These values can be explained by comparing the energies of the three systems. Figs. 11e and f reveal the energy in the elastic and rigid systems, respectively. Compared to the plastic fsp (11c), the internal energy the projectile contributes to the system for the elastic fsp is reduced by 86.6%, and for the rigid projectile it decreases to zero. Therefore, the internal energy of the concrete imparts more or all internal energy to the system than the projectile. For less energy dissipated by the projectile, the residual velocity of the FSP is increased, and a lower velocity is needed for complete perforation. Hence, using a projectile that has a greater deformation results in greater ballistic limit.

The experimental penetration tests were conducted on reinforced CORTUF; therefore, numerical simulations with fibers were also performed to determine the effects for varied fiber volumetric fractions (VF) of 3%, 4% and 5%. Whereas the average ballistic limit of the plain concrete was 1706 m/s, that of the fiber reinforced specimen was 1783 m/s. The energy–time plot for an impact velocity on a 3% VF fiber-reinforced specimen is shown in Fig. 11g. The energies show a very similar trend to the unreinforced specimens Fig. 11c with the clear difference being the energy dissipated in the fiber (DEF). However, most of the total dissipated energy (DE) is still in the concrete panel as the fiber dissipates only about 3% of the total dissipated energy.

Additionally, a parametric analysis was performed on Specimen A where three LDPM parameters were altered, tensile strength,
compressive strength, and the initial hardening modulus ratio, by plus and minus 15% to capture the variations exhibited in the experiments for a composite 15% stronger and weaker than the one calibrated. This value was selected because it closely captured the post-peak variation of the 3-point tests, and the other parameters were altered by the same quantity for consistency. Fig. 11h shows the effect of the varied LDPM parameters in terms of penetration depth versus velocity plots for the ultra-high-performance CORTUF specimen (UHPC), CORTUF 15% stronger (UHPC-U), CORTUF 15% weaker (UHPC-L), CORTUF with 3% and 5% VF (UHPC-F3, UHPC-F5), and a regular strength concrete (RSC). A typical RSC specimen was used with an experimental compressive strength of 38.35 MPa, an experimental tensile strength of 3.85 MPa, and a coarse aggregate size of 8 mm. The parameters used for these specimens are given in Table 2.

Fig. 11h demonstrates the variations in penetration depth for velocities below the ballistic limit of UHPC-F5 (1798 m/s). For the various CORTUF compositions, the penetration depth was only significantly altered for velocities close to the ballistic limit. The addition of fibers did not significantly affect the penetration depth of the specimen as the reinforced CORTUF consistently had a penetration depth within the scatter of the unreinforced CORTUF with strength 15% higher and lower than calibrated. Additionally, the weaker specimen was more affected by the increase in velocity, and the RSC had a greater penetration depth than the CORTUF. Moreover, a plot of exit versus strike velocities is shown in Fig. 11h for velocities above the ballistic limit. This plot follows a similar trend with greatest effect on the different CORTUF compositions, the penetration depth was only significantly altered for velocities close to the ballistic limit. We can conclude that, according to these simulations, varying material strength by plus/minus 15% has only an appreciable effect at about the ballistic limit, and the addition of fibers reduces the exit velocity. However, going from 3% to 5% fibers does not incur significant retardation on the exit velocity.

To determine which configuration best captured the failure patterns observed as well as to investigate the sensitivity of the penetration depth and perforation velocity to the applied constraints, the slabs were modeled using various boundary conditions. Three states were studied for specimen size A.

1. Fixed Edge — only nodes along the edges of the panel were selected, and they were fully constrained for displacement and rotation in all directions (see Fig. 12a).
2. Fixed Side — all nodes along the width of the panel sides were fixed to prevent any displacement or rotation (see Fig. 12b). This is the boundary condition applied to the panel for the previous simulations performed.
3. Free — no constraint was applied on the panel.

Fig. 12c shows the depth of penetration with time plot for an impact velocity of 1676 m/s. Penetration is first completed by the projectile impacting the free panel, whereas it takes the longest for the fixed-side. The fixed-edge condition, most similar to the given boundary conditions, closely follows the fixed side. However, for the time given, the difference in penetration depth of the free- and the fixed-sided panel is only about 5%. The reason the penetration depth is least in the fixed-side panel is due to the confinement the boundary provides for the panel, which cause resistance to the projectile. In reality, the actual effects on the boundary are non-uniform, and the fixed-side and completely free scenarios represent the limits bounding the true values of the boundary conditions.

The crack pattern for the RSC, UHPC, and UHPC with increasing fiber content were also explored using LDPM-F, and the results are displayed in Fig. 13. The first row of results shows the scabbing for the fixed-side boundary, the second row is the scabbing for the completely free-boundary condition, and the third row shows the side view for the fixed condition. Additionally, the first column represents the views for the RSC, the second column shows the UHPC, and the third and fourth columns show the UHPC 3% and 5% VF, respectively. The crater size increased for the UHPC in comparison to the RSC since the higher strength material with a smaller aggregate size has less inclusions to prevent the propagation of the crack through the specimen. Also, the scabbing effects are greatly reduced by the addition of fibers, but there is not much change as the VF increases from 3 to 5%. For the free-edged panels, splitting radial cracks, which propagate from the main damage zone outward as expected, are intensified for the more brittle UHPC and are reduced by the effect of the fibers.

Comparison of the panel failure with the actual experimental data is shown in Fig. 14. Experimental and numerical results of the impact face for the RSC is displayed in Fig. 14a and b, respectively. The red zone of the numerical results represents complete disintegration of the panel as seen in the experimental result. Figs. 14c and d show the experimental and numerical results for the UHPC-F3 impact face as well as the FSP penetrator damage. The crater size is reduced by the addition of the fibers in the numerical prediction and experimental results. However, the FSP penetrator obtained more damage than observed in the experimental results by modeling it as plastic. As previously investigated, this additional damage accrued to the projectile results in an overestimate of the ballistic limit and a decrease in residual velocity.

This information is also useful in evaluating the disparity between the numerical and experimental ballistic limit. The numerical ballistic limit of the plain concrete Size A was 1706 m/s, while the ballistic limit of the fiber reinforced specimen, tested by ERDC, was 1100 m/s and had a percent difference of 47% when
compared to the numerical ballistic limit for UHPC-F3 of 1783 m/s. Experiments performed on the panel resulted in perforation of the thinnest specimen, Size A, with 92.1% attenuation of the velocity, and 100% attenuation for the larger slabs, B and C, for an impact velocity of 1100 m/s. Based on experimental data on only one specimen, it is not possible to judge whether or not the difference between the numerical and experimental values are within the always-existing statistical scatter of experimental results. Additionally, many other effects contribute to the disparity of experimental and numerical ballistic limits, some of which were previously discussed; among them are the effects of a plastic projectile, boundary conditions of the panel, and the appreciable difference in penetration depth observed for impact velocities close to the ballistic limit for variations of CORTUF (see Fig. 11h and i). The effect of coarse graining the panels and modeling a coarse-grain size greater than 0.6 mm can also be a factor in determining the ballistic limit.

Further investigation was performed on the CORTUF-Plain specimen Sizes B and C to determine the effect on increasing panel depth, and average ballistic limits of 2042 m/s and 2469 m/s were obtained for these specimens, respectively. Compared to Specimen A with an average ballistic limit of 1706 m/s, increasing the depth of the panel by 25% resulted in a 20% increase in ballistic limit, and a panel with 50% increase in width leads to a 45% increase.

Fig. 13. Comparison of scabbing for fixed-side boundary, free boundary, and side crater view of (a) RSC (b) UHPC (c) UHPC-VF3 (d) UHPC-5.

Fig. 14. Projectile perforation comparison on impact face of NSC for a) experimental, and b) numerical tests; CORTUF specimens with deformed projectile for c) experimental, and d) numerical tests.
in ballistic limit. Fig. 15 shows a comparison of the three sizes, A, B, and C, for penetration depth versus velocity plots. For very low impact velocities, the penetration depth of each specimen is the same. However, as the velocities increase to the ballistic limit, the thinner specimens are able to perforate with a lower impact velocity. Additionally, Fig. 15b shows that the exit velocity for the thinnest specimen is greater than the larger specimens and have a similar trend as Panel B and C. Figs. 15c–f show the failure patterns observed for Panels B and C at their respective ballistic limits.

5. Conclusions

1. This paper successfully used LDPM-F to simulate the responses of the novel ultra-high-performance concrete, CORTUF, by first identifying the parameters through quasi-static experiments, single pull-out, uniaxial compression, uniaxial strain, triaxial compression and 3-point bending tests on CORTUF-Plain.
2. Without any change to previously identified parameters, LDPM-F was able to predict the following validation experiments for both plain and fiber-reinforced CORTUF: uniaxial compression, uniaxial strain, hydrostatic compression, triaxial compression (confined 10, 20, 50, 100, 200 and 300 MPa), 4-point bending, and 3-point bending (notched).
3. LDPM-F was also able to simulate vital testing conditions necessary to accurately reproduce the experimental results including the ability to capture both random and preferentially oriented fibers and cast or cored specimens.
4. The model remarkably captures the intrinsic transition of compressive behavior from a very brittle failure for zero or low confinement to a ductile failure for high confinement in the triaxial tests. Additionally, the model has the essential ability to capture localized cracks and characteristic crack patterns for plain concrete and a crack-bridging effect through fiber reinforcement for the CORTUF-Fiber model.
5. In-depth analysis of the failure mechanisms affecting CORTUF behavior revealed the importance of accounting for micro-splitting failure induced by the presence of hooks at fiber ends and the brittleness of the embedding matrix.
6. LDPM-F can be used confidently to investigate the behavior of CORTUF under impact loading, penetration and perforation events.
7. Simulations of projectile perforation showed the effect of increasing fiber content in the target specimens. Fibers added to the concrete reduce brittleness and hence the area of the crater, but it has only a secondary effect on reducing the exit velocity associated with strike velocities above the ballistic limit. Also, an increase in fiber content in excess of 3% does not seem to produce an appreciable improvement on the panel performance.
8. The reduced damaged area observed in CORTUF-Fiber compared to CORTUF-Plain is an important feature associated with post-impact residual strength of CORTUF panels and their performance under multiple hit scenarios.

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A. LDPM-F Constitutive Equations

A.1. LDPM Constitutive Equations

Normal and shear stresses in the elastic regime of LDPM are: 
\[ \sigma_N = E_N \varepsilon_N; \sigma_S = E_T \varepsilon_S; \alpha = E_{\alpha}, \] where \( E_N = E_0 \) effective normal modulus, \( E_T = aE_0 \) and \( \alpha = \) shear-normal coupling parameter.

For stresses and strains beyond the elastic limit, LDPM mesoscale failure is characterized by three mechanisms as described below.
Fracture and cohesion due to tension and tension-shear. For tensile loading \((\varepsilon > 0)\), the fracturing behavior is formulated through an effective strain, \(\varepsilon = \sqrt{\varepsilon_1^2 + \alpha (\varepsilon_M^2 + \varepsilon_L^2)}\), and stress, \(\sigma = \sqrt{\sigma_0^2 + (\sigma_M + \sigma_L^2)^2/\alpha}\), which define the normal and shear stresses as \(\sigma_N = \sigma_0 \sin(\varepsilon); \sigma_M = \alpha \sigma_0 \sin(\varepsilon); \sigma_L = \alpha \sigma_0 \cos(\varepsilon)\). The effective stress \(\sigma\) is incrementally elastic \((\varepsilon = E_0\varepsilon)\) and must satisfy the inequality \(0 < \varepsilon < \varepsilon_{\text{cr}}(t, \varepsilon)\), where \(\varepsilon_{\text{cr}} = \sigma_0 \exp[-H_0(\sigma)\varepsilon - \varepsilon_0(\varepsilon)]/\sigma_0(\varepsilon)\) and \(t\) is the maximum time, \(\varepsilon_0(\varepsilon) = \sqrt{\varepsilon}\). The post peak softening modulus is defined as \(H_0(\sigma) = H_0(\varepsilon_0(\varepsilon)/\varepsilon_0(\varepsilon))\), where \(H_0\) is the softening modulus in pure tension \((\varepsilon_0 = \varepsilon/2)\) expressed as \(H_0 = 2E_0(G_0 - 1); L = 2E_0G_0/\sigma_0^2\), \(L\) is the length of the tetrahedron edge; and \(G_0\) is the meso-scale fracture energy. LDPM provides a smooth transition between pure tension and pure shear \((\varepsilon = 0)\) with parabolic variation for stress given by \(\sigma_0(\varepsilon) = \sigma_0 t^2(\varepsilon - \sin(\varepsilon) + \sin^2(\varepsilon) + 4\cos^2(\varepsilon)/\sigma_0^2)/2\sigma_0^2\), where \(\sigma_0 = \sigma_0/\sigma_0\) is the ratio of shear strength to tensile strength.

Compaction and pore collapse from compression. Normal stresses for compressive loading \((\varepsilon < 0)\) are computed through the inequality \(-\varepsilon_0(\varepsilon) = \varepsilon_{\text{cr}}(t, \varepsilon)\), where \(\varepsilon_{\text{cr}} = \sigma_0(1 - \varepsilon_{\text{cr}})/H_0(\varepsilon_0(\varepsilon)/\varepsilon_0(\varepsilon))\), \(\varepsilon_0(\varepsilon) = \sqrt{\varepsilon}\), and \(\varepsilon\) is the plastic multiplier with loading-unloading conditions \(q < 0\) and \(\phi < 0\). The plastic potential is defined as \(\phi = \sqrt{\sigma_M^2 + \sigma_L^2} - \sigma_0(\sigma_N)\), where the nonlinear frictional law for the shear strength is assumed to be \(\theta_s = \sigma_0 + (\mu_0 - \mu_\sigma)\sigma_N - \mu_\sigma\sigma_N; \mu_0\) is the transverse normal stress; \(\mu_0\) and \(\mu_\sigma\) are the initial and final internal friction coefficients.

A.2 Fiber-Matrix Interaction Behavior

The pull-out resistance of the fiber is related to the load, \(P\), and the relative displacement/slipage, \(\delta\), between concrete and fiber. The full debonding is represented by a critical slippage load \(P_d = (2\sigma_d L^2/E_d)/\delta\) and \(\delta = L_0/2\) is a generic embedment length, \(G_d\) is the bond friction energy, and \(\tau_0\) is a constant frictional stress. When \(\delta < \delta_d\) (debonding), \(P = \sigma_d (\varepsilon - \varepsilon_0)/\varepsilon_0(\varepsilon)\), where \(\varepsilon = \varepsilon_0(\varepsilon)/\varepsilon_0(\varepsilon)\) is the fiber orientation, and \(\varepsilon_0(\varepsilon)\) is a material parameter.

The snubbing model adopted in LDPM-F assumes that the fiber, in a perfectly flexible manner, wraps around the exit of the tunnel crack that was shortened by spalling. The change in the tensile load for the flexible tendon is expressed as \(P(t) = \exp(k_{sn}\delta)/P(\varepsilon)\) where \(P(t)\) is the summation of all slip-friction and debonding forces parallel to the embedment length, and \(k_{sn}\) is the snubbing parameter. For a \(P(t)\) leading to fiber rupture, there will be no contribution from the fibers to structural strength and ductility. Single fiber pull-out tests on polyvinyl alcohol (PVA) revealed lower rupture loads for increasing \(\varepsilon\). This strength reduction (due to bending) is adopted in LDPM-F, and expressed as \(\sigma_{fr} \leq \sigma_{fr} e^{-k_{sn}\delta}\), where \(\delta\) is the axial stress in the crack-bridging segment, \(k_{sn}\) is a material parameter and \(\delta\) is the ultimate tensile strength of the fiber.

A detailed description of the LDPM idealization of concrete meso-structure can be found in Cusatis et al. [85] and more information on the fiber stiffness and unloading adopted in LDPM-F is provided in Ref. [88].

References


