Lattice Discrete Particle Model for Fiber-Reinforced Concrete. II: Tensile Fracture and Multiaxial Loading Behavior

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Abstract: In Part I of this two-part study, a theory is provided for the extension of the lattice discrete particle model (LDPM) to include fiber reinforcing capability. The resulting model, LDPM-F, is calibrated and validated in the present paper by comparing numerical simulations with experimental data gathered from the literature. The analyzed experiments include direct tension, confined and unconfined compression, and notched three-point bending tests. DOI: 10.1061/(ASCE)EM.1943-7889.0000392. © 2012 American Society of Civil Engineers.

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Introduction

In this two-part study, a discrete mesoscale model for heterogeneous materials, the lattice discrete particle model (LDPM), developed by Cusatis et al. (2011a) is extended to incorporate the effects of fiber reinforcing. In Part I (Schauffert and Cusatis 2012), the theoretical basis is provided. The formulation of LDPM is summarized, and details of the previously established (Yang et al. 2008) micromechanical fiber pullout model adapted for this study are presented. Part II addresses the calibration and validation of the fully formulated model, referred to as the LDPM-F. Presented first is a discussion of the LDPM-F model parameters, including the typical value ranges reported in the literature. For parameters in which pertinent information is not available, a justification is provided for the use of the recommended values. Then, the simulation of the experimental test data is summarized and conclusions are drawn. The analyzed data that are relevant to typical fiber-reinforced concrete (FRC) include direct tension, confined and unconfined compression, and notched three-point bending test results. For the simulations, LDPM-F was numerically implemented into the MARS software (Pelessone 2006), which is a structural analysis computer code with an object-oriented architecture that makes the implementation of new computational technologies very effective. In general, all of the definitions, notations, and abbreviations established and utilized in Part I will be retained here for Part II.

Discussion of Model Parameters

LDPM-F response depends on two sets of parameters: (1) LDPM material parameters governing the behavior of plain concrete; and (2) parameters governing the fiber-matrix interaction model; i.e., the behavior of one single fiber and its interaction with the surrounding matrix. A detailed discussion of the calibration procedure for the LDPM parameters can be found in Cusatis et al. (2011b).

The fiber-matrix interaction model, coupled with the LDPM framework in Part I, depends on nine material parameters: (1) bond fracture energy \( G_b \); (2) bond frictional stress \( \tau_0 \); (3) slip hardening-softening parameter \( \beta \); (4) spalling parameter \( k_{sp} \); (5) snubbing parameter \( k_{sn} \); (6) fiber tensile strength \( f_{tf} \); (7) fiber strength decay parameter \( k_{rup} \); (8) fiber elastic modulus \( E_f \); and (9) plastic deformation factor \( \gamma_p \). Accurate calibration of these parameters requires a significant amount of experimental data on fiber behavior, including the results of single fiber pullout testing with both straight and inclined fibers. Typically, experimental studies provide a very limited amount of this information. Therefore, model calibration often must be performed through indirect parameter identification based on the simulation of the structural response of experimental test specimens. However, even in this case the available experimental data are typically insufficient for the calibration of all parameters. To provide guidance regarding reasonable values that can be adopted for LDPM-F simulations, the following paragraphs summarize relevant information reported in the literature.

Bond Fracture Energy

The bond fracture energy between the fiber and matrix can vary widely. The interface between polyvinyl alcohol (PVA) and cementitious mortar is considered to have relatively high bonding...
(Zhang and Li 2002). This is attributed to some degree of chemical adhesion. Based on single fiber pullout tests from a cementitious composite, the range of reported $G_d$ values is approximately 4–6 N/m (Lin et al. 1999; Redon et al. 2001).

For the steel/mortar interface, some authors report the bond fracture energy to be relatively low and assume it to be zero (Leung and Geng 1998; Zhang and Li 2002). On the contrary, Shannag et al. (1997) reported a $G_d$ range of 6–12 N/m for smooth steel wire in a conventional cementitious mortar. Also, other studies (Naaman et al. 1991; Leung and Shapiro 1999) show steel fiber pullout curves with the characteristic sudden drop of load associated with full debonding and the start of the frictional phase, in which this phenomenon is attributed to a transition from a higher value of debonding friction to a lower value of basic pullout friction, rather than considering it to be a fracturing process governed by a fracture energy.

**Bond Frictional Stress**

Regarding the basic frictional resistance $\tau_0$ for smooth steel fibers in a normal strength, stress-free matrix, the reported range is approximately 1.0–4.5 MPa (Lim et al. 1987; Naaman and Najm 1991; Shamag et al. 1997). For steel fibers that have hooked ends or surface deformations, or that are crimped along the length, the reported frictional values tend to be higher, with ranges between 2 and 8 MPa (Lim et al. 1987; Banthia and Trotter 1994; Naaman and Najm 1991). The frictional resistance of PVA tends to be similar to that of smooth steel with $\tau_0$ ranges reported in the literature within 2.0–5.0 MPa (Kanda and Li 1998; Lin et al. 1999; Redon et al. 2001).

Most single fiber pullout testing is performed using specimens of plain, unstressed mortar. However, frictional resistance can be significantly affected by the environment in which an embedded fiber is located. Shamag et al. (1997) report that for steel fibers, pullout resistance increases as the fiber content of the mortar specimen increases. Also, Leung and Geng (1995) applied varying amounts of uniaxial compression in a direction orthogonal to the pulled fiber. Their results showed that the initial value of frictional resistance (i.e., immediately after debonding) can increase on the order of 1.5 MPa for each 10 MPa of applied lateral compression.

**Slip Hardening-Softening Parameter**

Similar to debonding, the evolution of the frictional pullout can vary widely. For synthetic fibers, such as PVA, interfacial friction can increase with slippage; i.e., slip hardening (Wang et al. 1988). This is typically attributed to fiber surface abrasion coming into increasing contact with the relatively unyielding surface of the cementitious tunnel crack. The values of $\beta$ for PVA can vary significantly owing to distinct differences in fiber surface texture (Redon et al. 2001). Recommendation of a typical value of $\beta$ would be inappropriate, and for PVA this parameter must be calibrated in accordance with the experimental evidence.

No experimentally determined values of $\beta$ for steel fiber are known to exist at this time. For smooth steel fibers, the literature suggests that interfacial friction typically decreases as pullout progresses; i.e., slip softening (Wang et al. 1988; Naaman et al. 1991). This behavior can be attributed to the relatively strong fiber breaking through the surface deformities along the cementitious tunnel. For steel fibers fabricated with a rough surface texture or hooked ends, there is experimental evidence (Banthia and Trotter 1994) indicating a very gradual—or even a negligible—decline in friction during pullout, thus indicating $\beta$ values near zero.

**Spalling Parameter**

To provide an approximate and generally applicable calibration for the spalling model [see Part I, Eq. (7)], a review of the literature for matrix spalling data was undertaken in Schauffert (2010). Although a number of studies were found that included single fiber pullout testing on fibers with varying embedment orientations $(0 < \theta < \pi/2)$, only three provided enough information for an assessment of the model (Li et al. 1990; Katz and Li 1995; Leung and Shapiro 1999). Analyses made in Schauffert (2010) concluded that a value of $k_w \sigma_t \approx 0.6f_c^\prime$, in which $f_c^\prime$ is the macroscopic, unconfined compressive strength, provides a reasonable estimate of slipp length for fibers oriented with low-to-moderate embedment angles $(\theta < 70^\circ \pm)$. For fibers oriented with higher embedment angles $(\theta > 70^\circ \pm)$, the model will likely underestimate spall lengths.

**Snubbing Parameter**

In the original study in which the snubbing model was first proposed (Li et al. 1990), $k_m$ was calculated to be approximately 1.0 and 0.7 for nylon and polypropylene fiber, respectively. For PVA, Lin et al. (1999) noted a generally accepted range of 0.5–0.9, and used 0.5 for the investigation of two specific PVA types. For this present study, simulations of PVA-reinforced concrete assume $k_m = 0.6$.

Regarding steel, this issue is not as straightforward. Data for steel fiber peak load versus embedment angle can be found in the literature (Naaman and Shah 1976; Banthia and Trotter 1994). However, detailed spalling information is required to make calculations similar to Li et al. (1990), and this was not provided in the noted studies. Even if reasonable estimates of spalling are assumed, the evidence indicates that the snubbing effect is not as strong for steel fibers compared with PVA. Therefore, this study has adopted a lower value of $k_m = 0.4$ for steel fiber-reinforced concrete (SFRC) simulation.

**Fiber Tensile Strength**

Experimental studies usually report the value of ultimate tensile strength $\sigma_{uf}$. However, some SFRC investigations do not report fiber mechanical properties. Based on the review of scientific and commercial literature undertaken for this present study, a large majority of references specify $\sigma_{uf}$ for steel fibers and wires in a range of 800–1,200 MPa. A fairly typical value of $\sigma_{uf} = 1,000$ is adopted here for SFRC simulations in which $\sigma_{uf}$ is not reported.

**Fiber Strength Decay Parameter**

For strength reduction, Kanda and Li (1998) determined a value of $k_{rup} = 0.3$ for PVA fibers and no substantially different recommendations were found in the literature. As noted in Part I, this study assumes that the ultimate strength reduction factor for steel fiber accounts for the likelihood of significant bending stresses for fibers inclined relative to the crack plane. For this initial version of LDPM-F, a $k_{rup}$ value was desired that acknowledged the existence of significant levels of bending stress but was not overly conservative. The SFRC simulations summarized in the subsequent section utilized $k_{rup} = 0.6$. This is twice the value used for PVA. For a specific example of a significantly inclined fiber, consider a fiber embedded at 60° from the crack face normal, and with ultimate tensile strength $\sigma_{uf} \approx 1,000$ MPa. Eq. 9 in Schauffert and Cusatis (2012) would return an apparent tensile strength of 533 MPa. The strength reduction of 467 MPa is commensurate with a common value of 400 MPa for steel reinforcing yield stress and, thus, this would approximately account for the possible existence of plastic deformation caused by bending.

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**Fiber Elastic Modulus**

Typically, $E_f$ is reported in experimental investigations of synthetic fibers. Where not reported in an SFRC investigation, the very common value of $E_f = 200$ GPa is reasonably assumed for steel fiber.

**Plastic Deformation Factor**

As discussed in Part I, $\gamma_p$ is relevant to SFRC simulations. Its determination must be based on the fitting of a numerical response to the experimental data used for calibration.

**Calibration and Validation of LDPM-F**

LDPM-F is validated through the comparison of numerical simulations with experimentally determined structural responses. Typical laboratory-sized structural test configurations are examined, including direct tension, multiaxial compression, and three-point bending. Each experimental data set typically provides the structural response for a plain concrete specimen (fiber volume percentage $V_f = 0\%$) and several values of nonzero $V_f$. The LDPM concrete parameters are determined by fitting a simulation to the plain concrete response. These parameter values are shown in Table 1. The fiber model calibration is then performed by fitting a simulation to one of the nonzero $V_f$ response curves. For validation purposes, the numerical responses for other fiber contents, determined by altering only the fiber volume fraction $V_f$, are then generated, and the results are compared with the experimental curves.

The experimental data are variable in that sometimes only one specimen is tested, or the reported structural response is described as a representative sample from several tests. In some instances this cannot be determined, and rarely is the average of several tests provided. For the LDPM-F simulations, unless noted otherwise, three different random LDPM particle configurations were generated for each test configuration. Each of these also included a random generation for the fiber placement and, therefore, the numerically generated curves presented herein are the average response from three distinct mesostructural models.

The primary goal of this paper is to demonstrate that LDPM-F can be calibrated and provide predictive capabilities. A secondary issue is to assess whether this goal can be achieved using generally accepted values of the main fiber-matrix interaction parameters. These are defined as the bond fracture energy $G_d$, the bond friction stress $\tau_0$, and the slip-hardening/softening parameter $\beta$. If so, this would then provide further validation of the model, including the mechanics of the parallel coupling between the LDPM concrete and the fiber-matrix interaction model. To be able to make useful conclusions regarding this goal, the other parameters were maintained at the values recommended in the previous section and were not varied arbitrarily to improve the data fitting process.

**Direct Tension**

A simulation of the fiber effect on the tensile behavior of concrete is shown in Figs. 1(a) and 1(b) for steel and PVA fibers, respectively. The experimental data were reported in Li (1998). Cross-sectional rectangular specimens (100 mm by 20 mm) were subjected to displacement-controlled direct tension. The concrete had a maximum aggregate size of 10 mm and an average compressive strength of 50 MPa. Several fiber configurations were investigated, and the results for the following two types were simulated: (1) steel with smooth circular surface, hooked ends, $d_f = 0.5$ mm, $L_f = 30$ mm, $E_f = 200$ GPa, and $\sigma_{df} = 1000$ MPa; and (2) PVA with $d_f = 0.66$ mm, $L_f = 30$ mm, $E_f = 29$ GPa, and $\sigma_{df} = 900$ MPa. Four different fiber volume percentages were tested: 2, 3, 6, and 8%. The structural responses are reported in terms of macroscopic average stress, or nominal stress (applied load divided by cross-sectional area), and macroscopic average strain, or nominal strain, which was computed using a combination of strain gauges with an original length of 120 mm.

Fig. 1(a) shows that the experimental behavior gradually transitions from softening for plain concrete and low $V_f$ to hardening and significant ductility for higher $V_f$. LDPM-F was calibrated by fitting a simulation to the $V_f = 6\%$ curve. The fiber parameters

### Table 1. LDPM Concrete Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Direct tension</th>
<th>Three-point bending</th>
<th>Uniaxial compression</th>
<th>Biaxial compression</th>
<th>Triaxial compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$ (GPa)</td>
<td>50</td>
<td>51</td>
<td>39</td>
<td>44</td>
<td>16</td>
</tr>
<tr>
<td>$\alpha$ (MPa)</td>
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<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_t$ (MPa)</td>
<td>4.5</td>
<td>2.21</td>
<td>3.5</td>
<td>3.9</td>
<td>2.43</td>
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<tr>
<td>$l_t$ (mm)</td>
<td>1,000</td>
<td>900</td>
<td>100</td>
<td>250</td>
<td>370</td>
</tr>
<tr>
<td>$G_t$ (N/m)</td>
<td>203</td>
<td>43</td>
<td>16</td>
<td>43</td>
<td>68</td>
</tr>
<tr>
<td>$\sigma_f/\sigma_t$</td>
<td>0.4</td>
<td>2.5</td>
<td>2.94</td>
<td>1.8</td>
<td>1.45</td>
</tr>
<tr>
<td>$n_t$</td>
<td>0.6</td>
<td>0.2</td>
<td>0.19</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_0$ (MPa)</td>
<td>100</td>
<td>195</td>
<td>77</td>
<td>70</td>
<td>50</td>
</tr>
<tr>
<td>$H_0/E_0$</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td>0.95</td>
<td>0.9</td>
</tr>
<tr>
<td>$\kappa_0$</td>
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<td>2.7</td>
<td>5</td>
<td>1.13</td>
<td>1.38</td>
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<tr>
<td>$\kappa_1$</td>
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<td>1</td>
<td>1</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
<td>0.4</td>
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<tr>
<td>$\mu_0$</td>
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<td>0.65</td>
<td>0.41</td>
<td>0.23</td>
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<tr>
<td>$\mu_\infty$</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.14</td>
<td>0.18</td>
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<tr>
<td>$\sigma_{30}$ (MPa)</td>
<td>50</td>
<td>600</td>
<td>50</td>
<td>91</td>
<td>50</td>
</tr>
<tr>
<td>$d_d$ (mm)</td>
<td>10</td>
<td>22</td>
<td>9</td>
<td>9.5</td>
<td>9.5</td>
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<tr>
<td>$d_0$ (mm)</td>
<td>3</td>
<td>10</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$c$ (kg/m$^3$)</td>
<td>470</td>
<td>357</td>
<td>413</td>
<td>405</td>
<td>410</td>
</tr>
<tr>
<td>$w/c$</td>
<td>0.45</td>
<td>0.49</td>
<td>0.46</td>
<td>0.6</td>
<td>0.58</td>
</tr>
<tr>
<td>$n_F$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
The reinforcement. Again, the fiber parameters were determined using the steel fiber simulations are reported for three in which the contours of the mesoscale crack openings at the end of $V_f$ values. For plain concrete, the crack pattern is characterized by one localized crack. Material above and below has unloaded owing to the strong softening in the structural response [Fig. 1(a)], and the diffuse microcracking has disappeared in these regions. For $V_f = 2\%$, there is still one main crack propagation. However, the entire specimen features diffuse cracking and no unloading is occurring. The absence of unloading outside of the main crack results from the fact that even though the overall behavior is mild softening [Fig. 1(a)], the structural response curve shows a nonzero residual stress associated with the fiber crack-bridging effect. Finally, for $V_f = 6\%$, the crack pattern is characterized by several branched cracks in which the propagation is arrested by the fiber effect. No unloading occurs outside the main cracks because the overall behavior is strain hardening up to a displacement corresponding to a nominal strain of 0.60\%. The crack patterns obtained in the numerical simulations are in excellent agreement with the published experimental evidence (Li et al. 1998).

**Three-Point Bending**

A simulation of the fiber effect on the three-point bending behavior of concrete is shown in Fig. 2(a). The experimental data were reported in Buratti et al. (2008). Notched specimens with a square cross sections of 150 mm by 150 mm and a span of 500 mm were subjected to displacement-controlled loading at midspan [Fig. 2(b)]. The notch depth was 25 mm, and its width was 4 mm. In such experiments, damage typically occurs only in a small region of the beam centered about the midspan notch. Therefore, to reduce the computational cost of the simulations, the ends of the beams were modeled with standard elastic finite elements, and the midspan portion was modeled with LDPM-F [Fig. 2(b)].

The concrete had a maximum aggregate size of 22 mm and an average compressive strength of 42 MPa. The PVA fibers were investigated with $d_f = 0.83 \text{ mm}$, $L_f = 40 \text{ mm}$, $E_f = 11.3 \text{ GPa}$, and $\sigma_{ut} = 600 \text{ MPa}$. Two different fiber volume percentages were tested: 0.4 and 0.8\%. The experimental results were reported in the form of load versus crack-mouth opening displacement (CMOD) curves and are shown in Fig. 2(a). In this case, the reported curves represent the average of two responses for the plain concrete and six responses for each fiber content.

LDPM-F was calibrated by fitting a simulation to the $V_f = 0.4\%$ curve, and the fiber parameters determined from that fit, $G_d = 5 \text{ N/m}$, $\tau_0 = 1.5 \text{ MPa}$, and $\beta = 0$, were in accordance with the experimental evidence previously summarized. Numerical prediction of the higher $V_f$ response was obtained by changing only that value to 0.8\%. Similar to the case of direct tension, LDPM-F was able to predict very well the response for the higher fiber content. The error, compared with the experimental results, was only a small percentage at the peak and initial postpeak and remained less than 20\% even in the far postpeak.

The numerical simulations are further investigated in Fig. 2(c), in which the contours of the mesoscale crack openings, relevant to CMOD = 0.8 mm, are reported for both plain and FRC. Because of the presence of the notch and the associated stress concentration, one main macroscopic crack developed in all three cases. However, this crack was the result of a band of mesoscale cracking characterized by a certain width. This result is in general agreement with previously reported experimental data and with the theoretical interpretation by Bažant and Oh (1983). For plain concrete, the crack was very localized, and in the vicinity of the notch tip the crack bandwidth was approximately 20 mm. This is commensurate with

![Fig. 1. Simulation results: (a) direct tension (steel fiber); (b) direct tension (PVA fiber); (c) mesoscale crack patterns (steel fiber)](image_url)
the maximum aggregate size, suggesting very little crack-bridging effect, and is in accordance with the nearly complete loss of load-carrying capacity indicated in Fig. 2(a) for $V_f = 0\%$ and CMOD = 0.8 mm. For FRC, the width of the crack band increased significantly with increasing fiber volume fraction. For $V_f = 0.8\%$, the crack bandwidth was approximately 55 mm, nearly 1.5 times the maximum aggregate size and nearly twice the fiber length. LDPM-F simulated correctly not only the increased load-carrying capacity but also the true mesoscale mechanisms that produced it.

**Uniaxial Compression**

Fig. 3(a) shows the fiber effect on the response of concrete to uniaxial compression. The experimental data were reported in Ezeldin and Balaguru (1992), in which cylindrical specimens (100-mm diameter by 200-mm height) were subjected to displacement-controlled compression. The experimental results are reported in terms of nominal stress versus nominal strain (based on the entire specimen length). The concrete had a maximum aggregate size of 9 mm and an average compressive strength of 36 MPa. Several fiber configurations were investigated, and the response curves for one were provided in the experimental report: steel with smooth circular surface, hooked ends, $d_f = 0.5$ mm, and $L_f = 30$ mm. The fiber mechanical properties were not reported. Three different fiber volume percentages were tested: $0.38$, $0.57$, and $0.76\%$. Two cylinders were tested for each $V_f$ value, and the response curves were reported for only one of the tested cylinders.

LDPM-F was calibrated by fitting the computational response to the $V_f = 0.57\%$ curve. The resulting parameter values were $G_d = 0$ N/m, $\tau_0 = 1.4$ MPa, and $\beta = -0.5$. The effective elastic modulus used was $\gamma_F E_f = 0.1(200) = 20$ GPa. The combination of $\gamma_F = 0.1$ and $\beta = -0.5$ enabled the calibration curve to fit both the peak load region and the far-right tail [Fig. 3(a)].

The experimental curves cross each other just beyond the peak-load region. The highest $V_f$ curve has the lowest peak load, and the lowest nonzero $V_f$ curve has the highest peak load. More intuitively, a positive correlation exists between postpeak load-carrying capacity and increasing fiber volume fraction. Several studies exist in the literature that have similar findings, including Mansur et al. (1999), Nataraja et al. (1999), and Bencardino et al. (2008). In general, these studies reported conflicting trends regarding compressive strength, with both mild increases and mild decreases associated with increasing fiber content. However, all studies reported an increase in postpeak load-carrying capacity and energy dissipation.

The LDPM-F simulations showed a mild increase in strength and a moderate increase in postpeak response with increasing $V_f$. For the postpeak response, the effect of the fibers was somewhat more pronounced in the experiments. For example, at a strain of 1.3%, LDPM-F simulates correctly the load-carrying capacity for $V_f = 0.57\%$, overestimates it by 20% for $V_f = 0.38\%$, and underestimates it by 14% for $V_f = 0.76\%$. However, the LDPM-F simulations can still be considered satisfactory, especially because the report of only one response curve per $V_f$ value provides no information on the scatter of the experimental data. Regarding peak load, the discrepancy between the experiments and simulations could involve the increased amount of fiber interaction occurring for increasing fiber content. Such an effect is not included in LDPM-F, which considers each fiber’s behavior to be independent of the presence of other fibers. Clarification of this aspect certainly warrants further investigation both experimentally and computationally.

**Triaxial Compression**

The effect of fiber reinforcing on the response of concrete to triaxial compression is shown in Figs. 3(b) and 3(c). The experimental data were reported in Chern et al. (1992). Cylindrical specimens (54-mm diameter by 108-mm height) were cored from larger concrete prisms. Pressurized fluid was used to apply uniform compressive stress around the lateral surface. Displacement-controlled...
corresponding experimental values. Nominal stresses are approximately 15%, on average, below the nominal strains of approximately 2 and 7%, the computed numerical response tends to underestimate the experimental result. Between load-carrying capacity owing to the fiber effect is very small and 10-MPa confinement [Fig. 3(c)]. As a matter of fact, the increase in postpeak load-carrying capacity with increasing fiber content. LDPM-F also simulates somewhat correctly the fiber effect at postpeak load-carrying capacity for 0 MPa confinement. 

The LDPM concrete was calibrated by fitting simulations to the 0- and 40-MPa confinement plain concrete curves. The simulation of 10-MPa confinement on plain concrete was performed without parameter adjustments. Fig. 3(c) shows that the 10-MPa numerical response tends to underestimate the experimental result. Between nominal strains of approximately 2 and 7%, the computed numerical nominal stresses are approximately 15%, on average, below the corresponding experimental values.

The fiber parameters, $G_d = 0$ N/m, $\tau_0 = 10.0$ MPa, and $\beta = -0.3$, were determined by fitting a simulation to the 0 MPa, $V_f = 2\%$ curve. This fit utilized $\gamma_p E_f = 0.2(200) = 40$ GPa. The other FRC simulations were performed without parameter adjustments. LDPM-F simulates very well the effect of fiber for 0 MPa confinement with $V_f = 1\%$ [Fig. 3(b)]. The predictive capability of LDPM-F displayed here for uniaxial compression is better than that of the previous section. One reason is that here the experimental response curves show an increase in both peak load and postpeak load-carrying capacity with increasing fiber content.

When considered in relation to the plain concrete response, LDPM-F also simulates somewhat correctly the fiber effect at 10-MPa confinement [Fig. 3(c)]. As a matter of fact, the increase in load-carrying capacity owing to the fiber effect is very small and approximately the same in the numerical simulations and the experimental results, although the values of the computational curves are below the experimental ones due to the computational underestimation of the plain concrete response. At 40 MPa, the numerical simulations underestimated the fiber effect. The far-right tangent of the experimental response curves display an average increase in load-carrying capacity of 9 MPa for each 1% increase in $V_f$, whereas that increase was approximately 3 MPa at 10-MPa confinement.

The observation that the fiber effect in LDPM-F appears to be independent of confinement level raises the subject, discussed previously, of increased frictional resistance for fibers subject to remotely applied compressive stresses. In general, this phenomenon should be more prevalent in triaxial tests compared with uniaxial or biaxial compression. Then, considering the confinement level, higher pressures should produce corresponding increases of frictional resistance to slip. In addition, other phenomena could be affected. In the presence of three-dimensional compression, the snubbing effect could be stronger, or frictional pullout could change from slip softening to slip hardening. The spalling characteristics could be significantly different. The confining pressure could place the matrix surrounding a fiber in a preloaded state of compressive strain prior to pullout. To fracture and displace a portion of the matrix (spall), the bearing force of the fiber would first have to overcome this compressive strain and then overcome the tensile resistance of the matrix.

Finally, a general observation can be made regarding crack sliding in the presence of compressive normal stress. The experimental fibers could contribute to crack sliding resistance in this situation. If so, then the assumption that LDPM-F ignores such a fiber contribution could affect the ability to simulate experimental results for triaxial compression.

**Biaxial Compression**

Experimental data regarding the effect of fiber reinforcing on the strength envelope for biaxial compression were reported in Yin et al. (1989). Square prisms, $150 \times 150$ mm with 40-mm out-of-plane...
thickness, were subjected to load-controlled biaxial compression. Four different nominal stress ratios were investigated. Given as applied horizontal nominal stress divided by vertical nominal stress, the ratios were \( \sigma_h/\sigma_v = 0.0, 0.2, 0.5, \) and 1.0. The concrete had a maximum aggregate size of 9.5 mm and an average compressive strength \( f'_c = 38 \) MPa. The steel fibers investigated were slit steel fibers, made by slicing a thin steel sheet into strips. The fiber cross section was 0.25 mm by 0.56 mm, which resulted in an equivalent diameter of 0.42 mm. The fiber length was 25 mm, and two fiber volume percentages, 1 and 2%, were tested.

The fibers were reported to have an average tensile strength of 414 MPa. This is a low value for \( \sigma_{uf} \) and close to a value commonly given for the yield strength of steel reinforcing. The assumption was made that the 414 MPa represents the tensile yield strength and that the ultimate rupture strength is likely higher. Model calibration was performed by assuming \( \sigma_{uf} = 1,000 \) MPa; however, the effect of \( \sigma_{uf} \) was investigated through a sensitivity analysis as will be discussed subsequently.

Fig. 3(d) shows the experimentally determined strength envelopes. The data are normalized by the average uniaxial (\( \sigma_h/\sigma_v = 0.0 \)) strength of the plain concrete (\( f'_c = 38 \) MPa). The experimental results are somewhat inconsistent in that, although they generally show a strength increase for increasing fiber content, the only significant difference between the \( V_f = 1 \) and 2% envelopes is at \( \sigma_h/\sigma_v = 0.2 \). Because each experimental point is reported to be the average of only two tests, and as indicated by the well-known variability of concrete testing, the experimental data for any particular point may not have captured well the true average. Because the 1 and 2% curves are essentially the same, the process of calibrating with one and predicting the other is not possible in this case. The goal was to see if a reasonable set of fiber parameters would allow LDPM-F to simulate the \( V_f = 2\% \) envelope.

Fig. 3(d) also shows the LDPM-F simulations. They were obtained using \( G_d = 0 \) N/m, \( \tau_0 = 13.0 \) MPa, and \( \beta = 0 \). The 2% envelope is simulated well. However, two main issues need to be addressed: (1) the use of \( \sigma_{uf} = 1,000 \) MPa and (2) the fact that the calibrated value of \( \tau_0 \) is somewhat higher than typical values reported in the literature. The use of \( \sigma_{uf} = 1,000 \) MPa was essential for this calibration. A sensitivity analysis determined that the strength envelope expands as \( \tau_0 \) is increased up to the value of 13 MPa. However, above that value the strength envelope actually contracts because of fiber rupture. The envelope also contracts if \( \tau_0 = 13.0 \) MPa is maintained but \( \sigma_{uf} \) is reduced. These observations indicate that, unlike the cases analyzed in the previous sections, fiber rupture influences the biaxial response in compression.

Regarding \( \tau_0 \), the relatively high value (for smooth, straight fibers) is further evidence concerning increased frictional resistance for fibers located in compressive environments. For the case of non-zero horizontal compression, a commonly observed failure mode for plain concrete is a splitting fracture oriented nearly parallel to the plane of loading (Yin et al. 1989). Fibers oriented nearly perpendicular to the plane of loading help to arrest the growth of such a fracture, and these fibers will experience both \( \sigma_h \) and \( \sigma_v \) as lateral compression on their embedded ends. For visualization of this concept, Fig. 3(e) shows the mesoscale cracking determined by LDPM-F for \( \sigma_h/\sigma_v = 0.5 \) and \( V_f = 0 \) and 2%. LDPM-F accurately captured the typical failure mode for the plain concrete, in which the nearly vertical crack seen on the short side extends completely across the specimen to the other (hidden) short side. For \( V_f = 2\% \), Fig. 3(e) shows that the main effect of the fiber content is to prevent the dominant splitting crack from occurring, thereby increasing the strength.

Finally, the use of the equivalent diameter for fibers with a rectangular cross-section should be considered. For this data set, the numerical fibers have 18% less surface area than the experimental fibers and, thus, a corresponding increase in \( \tau_0 \) can be expected.

**Summary and Conclusions**

In this two-part study, LDPM was extended successfully to include the effect of randomly dispersed fibers. The fiber-matrix interaction is modeled by adapting a previously established micromechanical theory providing the fiber crack-bridging force as a function of the crack opening. For all fibers that intersect an LDPM facet, the crack-bridging forces are summed and assumed to act in parallel with the LDPM constitutive behavior. The formulated model, LDPM-F, is able to reproduce the fiber toughening mechanisms and the associated increases in macroscopic strength and ductility indicated by the results of experimental testing. Specific conclusions include the following.

1. The goal of modeling individual fibers as discrete mesostructural elements was successfully accomplished, preserving the discrete and multiscale character of LDPM.
2. For a wide variety of loading configurations, LDPM-F could be calibrated with fiber-matrix interaction parameter values within, or reasonably close to, generally accepted ranges established in the literature. Once calibrated, LDPM-F is able to predict well the changes in structural strength and ductility as a function of varying fiber volume percentage. More specifically:
   a. LDPM-F can simulate accurately fiber effects on tensile fracturing not only in terms of load versus deformation curves but, more remarkably, in terms of crack patterns and damage distribution at failure.
   b. LDPM-F can capture the increased ductility owing to the fiber effect on uniaxial (unconfined) compressive behavior.
   c. LDPM-F reproduces satisfactorily the effect of fibers under multiaxial loading conditions.

The analysis of the simulation results and the review of the physical evidence on fiber pullout found in the literature suggest a few issues that warrant future investigation. Most notable is whether the fiber-matrix interaction parameters should depend upon the stress field in which they are situated. Except for the radial pressure of a triaxial test, the magnitudes of remotely applied pressures are constantly changing during a test. In addition, fibers are randomly oriented to the various applied pressures. The pullout model could be updated to allow the locally applied parameters, such as slip-frictional resistance \( \tau_0 \), to be dependent on global properties such as a remotely applied pressure, or they could be linked to the state of stress calculated locally at the LDPM facet level. In addition, the current assumption that fibers do not contribute to crack sliding resistance in situations of compressive normal stress should be further investigated. Finally, further research is needed toward the formulation of more general fiber-interaction models that are able to better address some of the following issues: fiber nonlinear behavior; fine features of fiber geometry, such as hooked ends and other forms of curvature; fibers with noncircular cross...
sections; improved unloading-reloading behavior; and, possibly, the effects of fiber-fiber interactions.

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References


