

答案は，日本語で記しても良い。

1. Obtain the general solution of the following ordinary differential equation with constant coefficients,

$$\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = 0.$$

2. Obtain the general solution of the following partial differential equation by separating variables (変数分離),

$$\frac{\partial^2 u}{\partial x \partial y} - u = 0.$$

3. Find the mean (平均), standard deviation (標準偏差) and variance (分散) of the following data set. Indicate clearly any formulas used.

50.6, 50.9, 49.1, 51.3, 50.5, 49.7.

4. Find the eigenvalues (固有値) and normalized eigenvectors (固有ベクトル) of the following matrix,

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 11 & -\sqrt{3} \\ 0 & -\sqrt{3} & 9 \end{bmatrix}.$$

**Solutions:**

(1) The characteristic equation is  $r^3 - 5r^2 + 8r - 4 = 0$ , with roots  $r = 1, 2, 2$ . The general solution is  $y = c_1 e^x + (c_2 + c_3 x) e^{2x}$ .

(2) Solution is of the form  $u(x, y) = F(x)G(y)$ . Insert into PDE to obtain  $\frac{F'}{F} = \frac{G}{G'} = k$ , which yields  $F(x) = e^{kx}$ ,  $G(y) = e^{y/k}$ . The general solution is  $u(x, y) = A e^{(kx + y/k)}$ .

(3) The formulas are,

mean  $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$ , standard deviation  $s = \left[ \frac{1}{n-1} \sum_{j=1}^n (x_j - \bar{x})^2 \right]^{1/2}$  and variance is  $s^2$ .

For the given set of data  $\bar{x} = 50.35$ ,  $s = 0.809$ ,  $s^2 = 0.655$ .

(4) Eigenvalues are 4, 8, 12. Eigenvectors are  $\pm \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $\pm \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ \sqrt{3} \end{bmatrix}$ ,  $\pm \frac{1}{2} \begin{bmatrix} 0 \\ -\sqrt{3} \\ 1 \end{bmatrix}$ .